ON PACKING COLORING OF LADDER AND TRIANGULAR LADDER GRAPH FAMILIES

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Abstract

In this paper, we find the packing chromatic number for the middle graph, total graph, central graph, line graph of ladder graph and triangular ladder graph.

Keywords:

Packing coloring, Packing chromatic number, Middle graph, total graph, Central graph, Line graph, Ladder graph and Triangular Ladder graph.

Introduction

Graph is a pictorial representation of collection of objects that are linked by lines. The objects are called as vertices and the lines are called as edges. A proper coloring of a graph is an assignment of colors to its elements of a graph such that no two adjacent elements have the same color. An improper coloring of a graph is an assignment of colors to the elements of the graph such that adjacent elements may share a common color.

A packing k-coloring of a graph G is a mapping \( \pi: V(G) \rightarrow 1, 2, ..., k \) such that any two vertices of color I are at distance at least i+1. The concept of packing coloring comes from the area of frequency assignment in wireless networks and was introduced by Goddard et al. under the name broadcast coloring. The packing chromatic number \( \chi_p \) of a graph G is the smallest integer k for which G has packing k-coloring. The term packing chromatic number was introduced by Bresar. Goddard et al. proved that the packing coloring problem is NP complete for general graphs.

Let G be a graph with vertex set V(G) and edge set E(G). The Middle graph \( M(G) \) of G, denoted by M(G) is defined as follows: The vertex set M(G) is V(G)∪E(G). Two vertices x,y of M(G) are adjacent in M(G) in case one of the following holds: (i) x, y are in the E(G) and x, y are adjacent in G, (ii) x is in V(G) and x,y are incident in G.
Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The Total graph of $G$, denoted by $T(G)$, is defined as follows: The vertex set $T(G)$ is $V(G) \cup E(G)$. Two vertices $x, y$ of $T(G)$ are adjacent in $T(G)$ in case one of the following holds: (i) $x, y$ are in $V(G)$ and $x$ is adjacent to $y$ in $G$. (ii) $x, y$ are adjacent in $G$. (iii) $x$ is in $E(G)$ and $x, y$ are incident in $G$.

The Central graph $C(G)$ of a graph $G$ is obtained from $G$ by adding an extra vertex on each edge of $G$, and then joining each pair of vertices of the original graph which were previously non-adjacent.

The Line graph $L(G)$ of $G$ denoted by $L(G)$ is the graph whose vertex set is the edge set of $G$. Two vertices of $L(G)$ are adjacent whenever the corresponding edges of $G$ are adjacent.

The Ladder graph $L_n$ is defined by $L_n = P_n \times K_2$ where $P_n$ is a path with $n$ vertices, $\times$ denotes the Cartesian product and $K_2$ is a complete graph with two-vertices.

A Triangular ladder graph $T L_n$, $n \geq 2$ is a graph obtained from $L_n$ by adding the edges $u_i v_{i+1}$, $1 \leq i \leq n - 1$. The vertices of $L_n$ are $u_i$ and $v_i$. $u_i$ and $v_i$ are the two paths in the graph $L_n$ where $i = \{1, 2, \ldots, n\}$.

In this paper we compute the packing coloring of ladder and triangular ladder graph families.

2. Packing coloring of middle, total, central and line graph of ladder graph:

Theorem: 2.1

For any ladder graph $L_n$, the packing chromatic number of its middle graph is

$$\chi_p(M(L_n)) = 3n - 1, n \geq 2.$$ 

Proof:

Let the vertices $M(L_n)$ is $\{a_i: 1 \leq i \leq n\} \cup \{b_i: 1 \leq i \leq n\} \cup \{d_i: 1 \leq i \leq n\}$. By the definition of middle graph $b_i$ and $d_i$ subdivides the edges of $a_i a_{i+1}$ and rungs for $(1 \leq i \leq n)$ respectively.

Assign the following packing $3n - 1$ coloring to $V(M(L_n))$.

For $1 \leq i \leq n$, assign the color $c_1$ to $a_i$.

For $1 \leq i \leq n$, assign the color $c_{i+1}$ to $b_i$ and $d_i$.

Therefore $\chi_p(M(L_n)) = 3n - 1$.

Theorem: 2.2
For any ladder graph $L_n$, the packing chromatic number of its total graph is

$\chi_p(T(L_n))=4n-1, \forall n \geq 2$.

**Proof:**

Let the vertices of $T(L_n)$ is \{ $a_i: 1 \leq i \leq n$ \} $\cup$ \{ $b_i: 1 \leq i \leq n$ \} $\cup$ \{ $d_i: 1 \leq i \leq n$ \}.

By the definition of total graph $b_i$ and $d_i$ subdivides the edges of $a_i a_{i+1}$ and rungs $(1 \leq i \leq n)$ respectively.

Assign the following packing 4n-1 coloring to $V(T(L_n))$.

For $1 \leq i \leq n$, assign the color $c_1$ to $d_i$.

For $1 \leq i \leq n$, assign the color $c_{i+1}$ to $b_i$ and $a_i$.

Therefore $\chi_p(T(L_n))=4n-1$.

**Theorem 2.3**

For any ladder graph $L_n$, the packing chromatic number of its central graph is

$\chi_p(C(L_n))=2n+1, \forall n \geq 2$.

**Proof:**

Let the vertices of $C(L_n)$ is \{ $a_i: 1 \leq i \leq n$ \} $\cup$ \{ $b_i: 1 \leq i \leq n$ \} $\cup$ \{ $d_i: 1 \leq i \leq n$ \}.

By the definition of central graph $b_i$ and $d_i$ subdivides the edges of $a_i a_{i+1}$ and rungs for $(1 \leq i \leq n)$ respectively.

Assign the following packing 2n+1 coloring to $V(C(L_n))$.

For $1 \leq i \leq n$, assign the color $c_1$ to $b_i$ and $d_i$.

For $1 \leq i \leq n$, assign the color $c_{i+1}$ to $a_i$.

Therefore $\chi_p(C(L_n))=2n+1$.

**Theorem 2.4**

For any ladder graph $L_n$, the packing chromatic number of its line graph is

$\chi_p(L(L_n))=2n-1, \forall n \geq 2$.

**Proof:**

Let the vertices of $L(L_n)$ is \{ $b_i: 1 \leq i \leq n$ \} $\cup$ \{ $d_i: 1 \leq i \leq n$ \}.
Where $b_i$ is the vertex corresponding to the edge $a_i a_{i+1}$ and $d_i$ is the vertex corresponding the rungs of $L_n$, $(1 \leq i \leq n)$ respectively.

Assign the following packing $2n-1$ coloring to $V(L(L_n))$.

For $1 \leq i \leq n$, assign the color $c_1$ to $d_i$.

For $1 \leq i \leq n$, assign the color $c_{i+1}$ to $b_i$.

Therefore $\chi_p(L(L_n))=2n-1$.

3. Packing coloring of middle, total, central and line graph of triangular ladder graph.

**Theorem: 3.1**

For any triangular ladder graph $TL_n$, the packing chromatic number of its middle graph is $\chi_p(M(TL_n)) = 4n - 2$, $\forall n \geq 2$.

**Proof:**

Let the vertices of $M(TL_n)$ is \{ $a_i: 1 \leq i \leq n$ $\} \cup {b_i: 1 \leq i \leq n}$ $\cup$ $\{d_i: 1 \leq i \leq n\} \cup \{j_i: 1 \leq i \leq n-1 \}$.

By the definition of middle graph $b_i$ and $d_i$ subdivides the edges of $a_i a_{i+1}$ and the rungs for $(1 \leq i \leq n)$; And $j_i$ subdivides the slanting edges for $(1 \leq i \leq n-1)$ respectively.

Assign the following packing $4n-2$ coloring to $V(M(TL_n))$.

For $1 \leq i \leq n$, assign the color $c_1$ to $a_i$.

For $b_i$ & $d_i (1 \leq i \leq n)$, $j_i (1 \leq i \leq n-1)$ assign the color $c_{i+1}$.

Therefore $\chi_p(M(TL_n))=4n-2$.

**Theorem: 3.2**

For any triangular ladder graph $TL_n$, the packing chromatic number of its total graph is $\chi_p(T(TL_n)) = 5n - 2$, $\forall n \geq 2$.

**Proof:**

Let the vertices of $T(T(L_n))$ is \{ $a_i: 1 \leq i \leq n$ $\} \cup {b_i: 1 \leq i \leq n}$ $\cup$ $\{d_i: 1 \leq i \leq n\} \cup \{j_i: 1 \leq i \leq n-1 \}$.

By the definition of total graph $b_i$ and $d_i$ subdivides the edges of $a_i a_{i+1}$ and the rungs for $(1 \leq i \leq n)$; And $j_i$ subdivides the slanting edges $(1 \leq i \leq n-1)$ respectively.
Assign the following packing 5n-2 coloring to V(T(TL_n)).

For 1 \leq i \leq n, assign the color c_1 to d_i.

For a_i \& b_i (1 \leq i \leq n), j_i (1 \leq i \leq n - 1) assign the color c_{i+1}.

Therefore \( \chi_p(T(TL_n)) = 5n-2 \).

**Theorem: 3.3**

For any triangular ladder graph TL_n, the packing chromatic number of its central graph is \( \chi_{p}(C(TL_n)) = 2n+1, \forall \ n \geq 2 \).

**Proof:**

Let the vertices of C(TL_n) is \{a_i: 1 \leq i \leq n\} \cup \{b_i: 1 \leq i \leq n\} \cup \{d_i: 1 \leq i \leq n\} \cup \{j_i: 1 \leq i \leq n - 1\}.

By the definition of central graph b_i and d_i subdivides the edges a_i a_{i+1} and the rungs for (1 \leq i \leq n); And j_i subdivides the slanting edges (1 \leq i \leq n - 1) respectively.

Assign the following packing 2n+1 coloring to V(C(TL_n)).

For b_i & d_i (1 \leq i \leq n), j_i (1 \leq i \leq n - 1) assign the color c_1.

For 1 \leq i \leq n, assign the color c_{i+1} to a_i.

Therefore \( \chi_{p}(C(TL_n)) = 2n + 1 \).

**Theorem: 3.4**

For any triangular ladder graph TL_n, the packing chromatic number of its line graph is \( \chi_{p}(L(TL_n)) = 3n-2, \forall \ n \geq 2 \).

**Proof:**

Let the vertices of L(TL_n) is \{b_i: 1 \leq i \leq n\} \cup \{d_i: 1 \leq i \leq n\} \cup \{j_i: 1 \leq i \leq n - 1\}.

Where b_i is the vertex corresponding to the edge of a_i a_{i+1}, d_i is the vertex corresponding to the rungs of TL_n for (1 \leq i \leq n); And j_i is the slanting edges for (1 \leq i \leq n - 1) respectively.

Assign the following packing (3n-2) coloring to V(L(TL_n)).

For 1 \leq i \leq n, assign the color c_1 to d_i.

For b_i (1 \leq i \leq n), j_i (1 \leq i \leq n - 1) assign the color c_{i+1}. 
Therefore \( \chi_p(L(TL_n)) = 3n - 2 \).

**References**


