Bipolar Valued Fuzzy *d*-Ideals of *d*-algebra

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Abstract: In this paper, we introduce and study the concept of bipolar fuzzy *d*-ideal of d-algebra and we characterize bipolar fuzzy *d*-ideal to the crisp *d*-ideal. Further, we prove that every bipolar fuzzy *d*-ideal is a bipolar fuzzy subalgebra and converse need not be. Also, we prove that the homomorphic image and inverse image of a bipolar fuzzy *d*-ideal is a bipolar fuzzy *d*-ideal.

Key words: *d*-ideal, bipolar fuzzy set, level cut of a bipolar fuzzy set, bipolar fuzzy *d*-ideal

1. Introduction

The concept of fuzzy subsets of a set was introduced by Zadeh, L.A. [7] in 1965. After that, there are several kinds of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, etc. In fuzzy sets the membership degree of elements range over the interval [0,1]. In 1994, Zhang [8] introduced the concept of bipolar-valued fuzzy sets which is an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree (0,1] indicates that elements somewhat satisfy the property and the membership degree [-1,0) indicates that elements somewhat satisfy the implicit counter-property.

Naggers, J. and Kim, H.S. [5] introduced and studied the concept of *d*-algebra, which is another generalization of BCK-algebras and investigated relations between *d*-algebras and BCK-algebras. Further, they discussed ideal theory in *d*-algebra. After that, they introduced the concepts of fuzzy *d*-ideal in *d*-algebras. Recently, Mohana Rupa, SVD., Lakshmi Prasannam, V. and Bhargavi, Y. [4] introduced and studied the concept of bipolar fuzzy *d*-algebra. This paper is a sequel to our study.

In this paper, we introduce and study the concept of bipolar fuzzy *d*-ideal of *d*-algebra and we characterize bipolar fuzzy *d*-ideal to the crisp d-ideal. Further, we prove that every bipolar fuzzy *d*-ideal is a bipolar fuzzy subalgebra and converse need not be. Also, we prove that the homomorphic image and inverse image of a bipolar fuzzy *d*-ideal is a bipolar fuzzy *d*-ideal.

2. Preliminaries

In this section we recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1[5]: A nonempty set X with a constant 0 and a binary operation $*$ is called a d-algebra, if for all $x, y \in X$ it satisfies the following axioms:

1. $x * x = 0$
2. $0 * x = 0$
3. $x * y = 0$ and $y * x = 0 \Rightarrow x = y$.

We refer $x \leq y$ $if\ and\ only\ if$ $x * y = 0$.

Definition 2.2[6]: Let I be a non-empty subset of a d-algebra X, then I is called d-ideal of X if $(i). x * y \in I\ and\ y \in I, then\ x \in I$
$(ii). x \in I\ and\ y \in X, then\ x * y \in I.$

Definition 2.3[3]: Let X and Y be two d-algebras. A mapping $f: X \to Y$ is called a homomorphism if $f(x * y) = f(x) * f(y)$, for all $x, y \in X$.

Definition 2.4[7]: Let X be a non-empty set. A fuzzy subset μ of the set X is a mapping $\mu: X \to [0,1]$.

Definition 2.5[8]: Let X be the universe of discourse. A bipolar-valued fuzzy set μ in X is an object having the form $\mu = \{x, \mu^-(x), \mu^+(x)/x \in X\}$, where $\mu^- : X \to [-1, 0]$ and $\mu^+: X \to [0, 1]$ are mappings.
For the sake of simplicity, we shall use the symbol $\mu = (X; \mu^-, \mu^+)$ for the bipolar-valued fuzzy set $\mu = \{x, \mu^-(x), \mu^+(x)/x \in X\}$, and use the notion of bipolar fuzzy sets instead of the notion of bipolar-valued fuzzy sets.

Definition 2.6[8]: Let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy set and $s \times t \in [-1, 0] \times [0, 1]$, the sets $\mu_s^N = \{x \in X/\mu^-(x) \leq s\}$ and $\mu_t^P = \{x \in X/\mu^+(x) \geq t\}$ are called negative s-cut and positive t-cut respectively. For $s \times t \in [-1, 0] \times [0, 1]$, the set $\mu_{(s,t)} = \mu_s^N \cap \mu_t^P$ is called (s, t)-set of $\mu = (X; \mu^-, \mu^+)$.

Definition 2.7[3]: Let $f: X \to Y$ be a homomorphism from a set X onto a set Y and let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy set of X and $\sigma = (Y; \sigma^-, \sigma^+)$ be two bipolar fuzzy set of Y, then the homomorphic image $f(\mu)$ of μ is $f(\mu) = ((f(\mu))^-, (f(\mu))^+)$ defined as for all $y \in Y$

$$(f(\mu))^-(y) = \begin{cases} \max\{\mu^-(x)/x \in f^{-1}(y)\}, if\ f^{-1}(y) \neq \emptyset \\ 0,\ otherwise \end{cases}$$

and

$$(f(\mu))^+(y) = \begin{cases} \max\{\mu^+(x)/x \in f^{-1}(y)\}, if\ f^{-1}(y) \neq \emptyset \\ 0,\ otherwise \end{cases}$$

The pre-image $f^{-1}(\sigma)$ of σ under f is a bipolar set defined as $(f^{-1}(\sigma))^-(x) = \sigma^-(f(x))$ and $(f^{-1}(\sigma))^+(x) = \sigma^+(f(x))$, for all $x \in X$.

Definition 2.8[3]: Let μ be a fuzzy set of a d-algebra X. Then, μ is said to be fuzzy d-ideal of X if it satisfies for all $x, y \in X$

$(i).\ \mu(x) \geq \min\{\mu(x * y), \mu(y)\}$
$(ii).\ \mu(x * y) \geq \mu(x)$

3. Bipolar Fuzzy d-algebra

In this paper, we introduce and study the concept of bipolar fuzzy *d*-ideal of *d*-algebra and we characterize bipolar fuzzy *d*-ideal to the crisp *d*-ideal. Further, we prove that every bipolar fuzzy *d*-ideal is a bipolar fuzzy subalgebra and converse need not be. Also, we prove that the homomorphic image and inverse image of a bipolar fuzzy *d*-ideal is a bipolar fuzzy *d*-ideal.
Throughout this section X stands for a *d*-algebra unless otherwise mentioned.

Now, we introduce the following.

Definition 3.1: A Bipolar fuzzy set $\mu = (X; \mu^-, \mu^+)$ in X is called a bipolar fuzzy *d*-ideal if it satisfies the following properties: for any $x, y \epsilon X$,
$(i).\ \mu^-(x) \leq max\{\mu^-(x * y), \mu^-(y)\}$
$(ii).\ \mu^-(x * y) \leq \mu^-(x)$
$(iii).\ \mu^+(x) \geq \min\{\mu^+(x * y), \mu^+(y)\}$
$(iv).\ \mu^+(x * y) \geq \mu^+(x)$

Example 3.2: Consider a d-algebra $X = \{0, 1, 2\}$ with the following Cayley table

$*$	0	1	2
0	0	0	0
1	2	0	2
2	1	1	0

Define a bipolar fuzzy se $\mu = (X; \mu^-, \mu^+)$, where $\mu^-: X \to [-1, 0]$ and $\mu^+: X \to [0, 1]$ as

$$\mu^-(x) = \begin{cases} -0.7, & if\ x = 0 \\ -0.5, & if\ x = 1 \\ -0.4, & if\ x = 2 \end{cases} \text{ and } \mu^+(x) = \begin{cases} 0.9, & if\ x = 0 \\ 0.8, & if\ x = 1 \\ 0.6, & if\ x = 2 \end{cases}$$

Then μ is a bipolar fuzzy *d*-ideal.

Proposition 3.3: If $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy *d*-ideal of X, then $\mu^-(0) \leq \mu^-(x)$ and $\mu^+(0) \geq \mu^+(x)$, for all $x \in X$.

Proof: Let $x \in X$.
Now, $\mu^-(0) = \mu^-(x * x) \leq \mu^-(x)$ and $\mu^+(0) = \mu^+(x * x) \geq \mu^+(x)$.

Lemma 3.4: Let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy *d*-ideal of X. If $x * y \leq z$, then $\mu^-(x) \leq \max\{\mu^-(y), \mu^-(z)\}$ and $\mu^+(x) \geq \min\{\mu^+(y), \mu^+(z)\}$ for all $x, y, z \in X$.

Proof: Let $x, y, z \in X$ such that $x * y \leq z$.
Then $(x * y) * z = 0$
Now, $\mu^-(x) \leq \max\{\mu^-(x * y), \mu^-(y)\} \leq \max\{\max\{\mu^-((x * y) * z), \mu^-(z)\}, \mu^-(y)\} = \max\{\max\{\mu^-(0), \mu^-(z)\}, \mu^-(y)\} = \max\{\mu^-(z)\}, \mu^-(y)\}$ and
Also, $\mu^+(x) \geq \min\{\mu^+(x * y), \mu^+(y)\} \geq \min\{\min\{\mu^+((x * y) * z), \mu^+(z)\}, \mu^+(y)\} = \min\{\min\{\mu^+(0), \mu^+(z)\}, \mu^+(y)\} = \min\{\mu^+(z)\}, \mu^+(y)\}$.

Lemma 3.5: Let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy d-ideal of X. If $x \leq y$, then $\mu^-(x) \leq \mu^-(y)$ and $\mu^+(x) \geq \mu^+(y)$ for all $x, y \in X$.

Proof: Let $x, y \in X$ such that $x \leq y$.
Then $x * y = 0$.
Now, $\mu^-(x) \leq \max\{\mu^-(x * y), \mu^-(y)\} = \max\{\mu^-(0), \mu^-(y)\} \leq \mu^-(y)$.
Also, $\mu^+(x) \geq \min\{\mu^+(x * y), \mu^+(y)\} = \min\{\mu^+(0), \mu^+(y)\} \geq \mu^+(y)$.

Theorem 3.6: Let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy d-ideal of X, then for any $x, x_1, x_2, \dots, x_n \in X$ such that $(\dots((x * x_1) * x_2) * \dots * x_n) = 0$ implies $\mu^-(x) \leq \max\{\mu^-(x_1), \mu^-(x_2), \dots, \mu^-(x_n)\}$ and $\mu^+(x) \geq \min\{\mu^+(x_1), \mu^+(x_2), \dots, \mu^+(x_n)\}$.

Proof: Proof is clear by using lemma:3.4, 3.5 and induction on n.

Theorem 3.7: Every bipolar fuzzy d-ideal of X is a bipolar fuzzy subalgebra of X.

Proof: Let $\mu = (X; \mu^-, \mu^+)$ be a bipolar fuzzy d-ideal
Then $\mu^-(x * y) \leq \mu^-(x) \leq max\{\mu^-(x * y), \mu^-(y)\} \leq max\{\mu^-(x), \mu^-(y)\}$ and
$\mu^+(x * y) \geq \mu^+(x) \geq \min\{\mu^+(x * y), \mu^+(y)\} \geq min\{\mu^+(x), \mu^+(y)\}$.
Thus μ is a bipolar fuzzy subalgebra of X.

But every bipolar fuzzy subalgebra is not a bipolar d-ideal.

Example 3.8: Consider a d-algebra $X = \{0, 1, 2, 3\}$ with the following Cayley table

*	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	1	0	2
3	3	3	3	0

Define a bipolar fuzzy se $\mu = (X; \mu^-, \mu^+)$t, where $\mu^+ : X \to [0, 1]$ and $\mu^- : X \to [-1, 0]$ as

$$\mu^-(x) = \begin{cases} -0.8, & when\ x = 0,1,3 \\ -0.2, & when\ x \neq 0,2 \end{cases} \text{ and } \mu^+(x) = \begin{cases} 0.7, & when\ x = 0,1,3 \\ 0.3, & when\ x = 2 \end{cases}$$

Thus μ is a bipolar fuzzy subalgebra but not bipolar fuzzy d-ideal.

Theorem 3.9: A bipolar fuzzy set $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy ideal of X if and only if $\overline{\mu^-}$and $\mu^+$are fuzzy d-ideals of X.

Proof: Suppose $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy d-ideal of X.
Let $x, y \in X$.
Now, (i). $\overline{\mu^-}(x) = 1 - \mu^-(x) \geq 1 - \max\{\mu^-(x * y), \mu^-(y)\} = \min\{1 - \mu^-(x * y), 1 - \mu^-(y)\} = \min\{\overline{\mu^-}(x * y), \overline{\mu^-}(y)\}$
(ii). $\overline{\mu^-}(x * y) = 1 - \mu^-(x * y) \geq 1 - \mu^-(x) = \overline{\mu^-}(x)$.
Thus $\overline{\mu^-}$ is a fuzzy d-ideal of X.
Clearly by definition μ^+ is fuzzy d-ideal of X.
Conversely suppose that $\overline{\mu^-}$and $\mu^+$are fuzzy d-ideals of X.
Let $x, y \in X$.

$(i).\, \mu^-(x) = 1 - \overline{\mu^-}(x) \le 1 - \min\{\overline{\mu^-}(x*y), \overline{\mu^-}(y)\} = \max\{1 - \overline{\mu^-}(x*y), 1 - \overline{\mu^-}(y)\} = \max\{\mu^-(x*y), \mu^-(y)\}$.
$(ii).\, \mu^-(x*y) = 1 - \overline{\mu^-}(x*y) \le 1 - \overline{\mu^-}(x) = \mu^-(x)$
Thus $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy d-ideal of X.

Theorem 3.10: A bipolar fuzzy set $\mu = (X; \mu^-, \mu^+)$ of X is a bipolar fuzzy d-ideal of X if and only if the level cuts are d-ideals of X i.e., for all $s \times t \in [-1, 0] \times [0, 1]$, $\emptyset \ne \mu_s^N$ and $\emptyset \ne \mu_t^P$ are d-ideals of X.

Proof: Suppose $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy d-ideal.
Let $s \times t \in [-1,0] \times [0, 1]$ such that $\mu_s^N \ne \emptyset$ and $\mu_t^P \ne \emptyset$.
(I). Let $g * h, h \in \mu_s^N$.
That implies $\mu^-(g*h) \le s, \mu^-(h) \le s$.
Since $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy subalgebra, we have
$\mu^-(g) \le max\{\mu^-(g*h), \mu^-(h)\} \le s$
$\Rightarrow g \in \mu_s^N$.
(ii). Let $g \in \mu_s^N$ and $h \in X$.
That implies $\mu^-(g) \le s$.
Since $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy d-ideal, we have $\mu^-(g*h) \le \mu^-(g) \le s$.
$\Rightarrow g*h \in \mu_s^N$
Thus μ_s^N is a d-ideal of X.
(iii). Also, let $x*y, y \in \mu_t^P$.
That implies $\mu^+(x*y) \ge t \text{ and } \mu^+(y)\} \ge t$.
Since $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy d-ideal, we have
$\mu^+(x) \ge min\{\mu^+(x*y), \mu^+(y)\} \ge t$.
$\Rightarrow x \in \mu_t^P$.
(iv). Let $x \in \mu_t^P$ and $y \in X$.
That implies $\mu^+(x) \ge t$.
Since $\mu = (X; \mu^-, \mu^+)$ is a bipolar fuzzy d-ideal, we have $\mu^+(x*y) \ge \mu^+(x) \ge t$.
$\Rightarrow x*y \in \mu_t^P$
Therefore μ_t^P is a d-ideal of X.
Thus μ_s^N and μ_t^P are d-ideals of X.
Conversely suppose that the level cuts μ_s^N and μ_t^P are d-ideals of X, for all $s \times t \in [-1, 0] \times [0, 1]$.
Let $x, y \in X$ such that $\mu^-(x) > max\{\mu^-(x*y), \mu^-(y)\}$.
Take $s_0 = \frac{1}{2}(\mu^-(x) + \max\{\mu^-(x*y), \mu^-(y)\})$, where $s_0 \in [-1, 0]$.
That implies $\max\{\mu^-(x*y), \mu^-(y)\} < s_0 < \mu^-(x)$.
So, $x*y, y \in \mu_{s_0}^N$ and $x \notin \mu_{s_0}^N$.
Which is a contradiction to $\mu_{s_0}^N$ is a d-ideal.
Hence $\mu^-(x) \le max\{\mu^-(x*y), \mu^-(y)\}$.
Again let $x, y \in X$ such that $\mu^-(x*y) > \mu^-(x)$.
Take $s_0 = \frac{1}{2}(\mu^-(x*y), \mu^-(x))$.
That implies $\mu^-(x*y) < s_0 < \mu^-(x)$.
So, $x \in \mu_{s_0}^N$ and $x*y \notin \mu_{s_0}^N$.
Which is a contradiction to $\mu_{s_0}^N$ is a d-ideal.
Hence $\mu^-(x*y) \le \mu^-(x)$
Let $x, y \in X$ such that $\mu^+(x) < min\{\mu^+(x*y), \mu^+(y)\}$.

Take $t_0 = \frac{1}{2}(\mu^+(x) + \min\{\mu^+(x*y), \mu^+(y)\})$, where $t_0 \in [0,1]$.
That implies $\mu^+(x) < t_0 < \min\{\mu^+(x*y), \mu^+(y)\}$.
So, $x*y, y \in \mu^P_{t_0}$ and $x \notin \mu^P_{t_0}$.
Which is a contradiction to $\mu^P_{t_0}$ is a d-ideal.
Hence $\mu^+(x) \geq min\{\mu^+(x*y), \mu^+(y)\}$.
Again let $x, y \in X$ such that $\mu^+(x*y) < \mu^+(x)$.
Take $t_0 = \frac{1}{2}(\mu^+(x*y), \mu^+(x))$.
That implies $\mu^+(x*y) < t_0 < \mu^+(x)$.
So, $x \in \mu^P_{t_0}$ and $x*y \notin \mu^P_{t_0}$.
Which is a contradiction to $\mu^P_{t_0}$ is a d-ideal. Hence $\mu^+(x*y) \geq \mu^+(x)$
Thus $\mu = (X; \mu^-, \mu^+)$ of X is a bipolar fuzzy d-ideal of X.

Theorem 3.11: Let f be a homomorphism from a d-algebra X onto a d-algebra Y. Let σ be a bipolar fuzzy d-ideal of Y, then the pre-image $f^{-1}(\sigma)$ of σ is a bipolar fuzzy d-ideal of X.

Proof: Let $x, y \in X$.
Now,
$(i). (f^{-1}(\sigma))^-(x) = \sigma^-(f(x))$
$\leq \max\{\sigma^-(f(x*y)), \sigma^-(f(y))\}$
$= \max\{(f^{-1}(\sigma))^-(x*y), (f^{-1}(\sigma))^-(x)\}$
$(ii). (f^{-1}(\sigma))^-(x*y) = \sigma^-(f(x*y)) \leq \sigma^-(f(x)) = (f^{-1}(\sigma))^-(x)$
$(iii). (f^{-1}(\sigma))^+(x) = \sigma^+(f(x))$
$\geq \min\{\sigma^+(f(x*y)), \sigma^+(f(y))\}$
$= \min\{(f^{-1}(\sigma))^+(x*y), (f^{-1}(\sigma))^+(x)\}$
$(iv). (f^{-1}(\sigma))^+(x*y) = \sigma^+(f(x*y)) \geq \sigma^+(f(x)) = (f^{-1}(\sigma))^+(x)$.
Thus $f^{-1}(\sigma)$ is a bipolar fuzzy d-ideal of X.

Theorem 3.12: Let f be a homomorphism from a d-algebra X onto a d-algebra Y. Let μ be a bipolar fuzzy d-ideal of X, then the homomorphic image $f(\mu)$ of μ is a bipolar fuzzy d-ideal of Y.

Proof: Let $x, y \in Y$.
Suppose neither $f^{-1}(x)$ nor $f^{-1}(y)$ is non-empty.
since f is homomorphism and so there exist $a, b \in X$ such that $f(a) = x$ and $f(b) = y$ it follows that $a*b \in f^{-1}(x*y)$.
Now,
$(i). (f(\mu))^-(x) = \max\{\mu^-(z)/z \in f^{-1}(x)\}$
$\leq \max\{\max\{\mu^-(a*b), \mu^-(b)\}/a \in f^{-1}(x), b \in f^{-1}(y)\}$
$= \max\{\max\{\mu^-(a*b)/a \in f^{-1}(x)\}, \max\{\mu^-(b)/ b \in f^{-1}(y)\}\}$
$= \max\{(f(\mu))^-(x*y), (f(\mu))^-(y)\}$
$(ii). (f(\mu))^-(x*y) = \max\{\mu^-(z)/z \in f^{-1}(x*y)\}$
$\leq \max\{\mu^-(a*b)/a \in f^{-1}(x),\ b \in f^{-1}(y)\}$
$\leq \max\{\mu^-(a)\}/a \in f^{-1}(x)\}$
$= (f(\mu))^-(x)$
$(iii). (f(\mu))^+(x) = \max\{\mu^+(z)/z \in f^{-1}(x)\}$
$\geq \max\{\min\{\mu^+(a*b), \mu^+(b)\}/a \in f^{-1}(x),\ b \in f^{-1}(y)\}$

$$= \min\{\max\{\mu^+(a*b)/a \in f^{-1}(x)\}, \max\{\mu^+(b)/\ b \in f^{-1}(y)\}\}$$
$$= \min\{(f(\mu))^+(x*y), (f(\mu))^+(y)\}$$

(iv). $(f(\mu))^+(x*y) = \max\{\mu^+(z)/z \in f^{-1}(x*y)\}$

$$\geq \max\{\mu^+(a*b)/a \in f^{-1}(x),\ b \in f^{-1}(y)\}$$
$$\geq \max\{\mu^+(a)\}/a \in f^{-1}(x)\}$$
$$= (f(\mu))^+(x)$$

Thus $f(\mu)$ is a bipolar fuzzy d-ideal of Y.

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