A New class of Neutrosophic Nano gb-closed sets in Neutrosophic Nano topological spaces

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Abstract
The purpose of this paper is to introduced a new class of neutrosophic nano generalized closed sets namely, neutrosophic nano gb-closed set in a neutrosophic nano topological spaces. Also we have introduced neutrosophic nano semi closed, neutrosophic nano α-closed, neutrosophic nano regular closed and investigate some of their properties.

Keywords:
Neutrosophic nano gb-closed set, Neutrosophic nano α - closed set, Neutrosophic nano semi closed, Neutrosophic nano g-closed, Neutrosophic nano sg-closed.

1. INTRODUCTION

Andrijevic[1] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. Levine [6] derived the concept of generalized closed sets in topological space. Al Omari and Mohd.Salmi Md.Noorani studied the class of generalized b-closed sets. The notation of Nano topology was introduced by Lellis Thivagar[4] which was defined in terms of approximations and boundary regions of a subset of an universe using an equivalent relation on it. Nano gb-closed set was initiated by Dhanis Arul Marry and I.Arockiarani[3]. Fuzzy and intuitionistic fuzzy were introduced by zadeh[9] and K.Atanassav[2]. The new theory neutrosophic set described by membership, indeterminacy and non-membership were introduced by smarandache[8]. The neutrosophic set in $(X,\tau_N(S))$ is having the form $S=\{\langle x,M_s(x),I_s(x),N_s(x)\rangle : x\in(X,\tau_N)\}$ where the functions $M_s(x)$, $I_s(x),N_s(x)$ denoted the degree of membership, indeterminacy, degree of non-membership. The neutrosophic set $S=\{\langle x,M_s(x),I_s(x),N_s(x)\rangle : x\in(X,\tau_N)\}$ is called a subset of $T=\{\langle x,M_T(x),I_T(x),N_T(x)\rangle : x\in(X,\tau_N)\}$ [in short $S\subset T$] if degree of membership and indeterminacy is minimum in S and degree of non-membership is maximum in S or degree of membership is minimum and degree of non-membership and indeterminacy is maximum in S.

The complement on NTS $S=\{\langle x, M_s(x), I_s(x), N_s(x)\rangle : x\in(X,\tau_N)\}$ is $S^c=\{\langle x, N_s(x), I_s(x), M_s(x)\rangle : x\in(X,\tau_N)\}$. Parimala et .al[7] introduced and studied the concept of neutrosophic nano closure. Now Lellis Thivagar et.al [5] explored a new concept of neutrosophic nano topology. In that paper he discussed about neutrosophic nano interior and neutrosophic nano closure.

In this paper, basic properties of neutrosophic nano closed, neutrosophic nano α -closed, neutrosophic nano pre-closed were introduced. It also established the notion of neutrosophic nano sg - closed set, neutrosophic nano g-closed and studied some of their properties.

2. PRELIMINARIES

Definition 2.1[4]: Let U be a non- empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Let X is a subset of U, then the lower approximation of X with respect to R is denoted by $\underline{R}=\bigcup_{x\in\cup}\{R(x): R(x)\subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x\in U$.

Definition 2.2[4]: The upper approximation of X with respect to R is the set of all object, which can be possibly classified as X with respect to R and it is denoted by $\overline{R}=\bigcup_{x\in\cup}\{R(x): R(x)\bigcap X\neq\phi\}$.

Definition 2.3[4]: The boundary region of X with respect to R is the set of all objects, which can be possibly classified as neither as X nor as not X with respect to R and it is denoted by $B_R=\overline{R}-\underline{R}$.

Definition 2.4[4]: If (U, R) is an approximation space and $X, Y\subseteq U.$ Then

1.$\underline{R}\subseteq X\subseteq\overline{R}$

2.$\underline{R}(\phi)=\overline{R}(\phi)=\phi$ and $\underline{R}(\bigcup)=\overline{R}(\bigcup)=\bigcup.$

3.$\overline{R}(X\cup Y)=\overline{R}(X)\cup\overline{R}(Y)$

4.$\overline{R}(X\cap Y)=\overline{R}(X)\cap\overline{R}(Y)$

5.$\underline{R}(X\cup Y)=\underline{R}(X)\cup\underline{R}(Y)$

6.$\underline{R}(X\cap Y)=\underline{R}(X)\cap\underline{R}(Y)$

7.$\overline{R}(X)\subseteq\overline{R}(Y)$ and $\underline{R}(X)\subseteq\underline{R}(Y)$ whenever $X\subseteq Y$

8. $\overline{R}(X^c)=(\underline{R})^c$ and $\underline{R}(X^c)=(\overline{R})^c$

9.$\underline{R}(\underline{R})=\overline{R}(\underline{R})=\underline{R}$

10.$\overline{R}(\overline{R})=\underline{R}(\overline{R})=\overline{R}$

Definition 2.5[5]: Let U be an universe and R be an equivalence relation on U and Let S be a neutrosophic subset of U. Then the neutrosophic nano topology is defined by $\tau_N(S)=\{0_N, 1_N, \underline{N}(S), \overline{N}(S), B_N(S)\}$, where

1.$\underline{N}(S)=\{<y,M_{\underline{R}(y)},I_{\underline{R}(y)},N_{\underline{R}(y)}>/z\in[y]_R,y\in U\}$

2.$\overline{N}(S)=\{<y,M_{\overline{R}(y)},I_{\overline{R}(y)},N_{\overline{R}(y)}>/z\in[y]_R,y\in U\}$

3.$B_N(S)=\overline{N}-\underline{N}$

$where M_{\underline{R}(y)}=\wedge_{z\in[y]R}M_S(z),I_{\underline{R}(y)}=\wedge_{z\in[y]R}I_S(z),N_{\underline{R}(y)}=\vee_{z\in[y]R}N_S(z),$

$M_{\overline{R}(y)}=\vee_{z\in[y]R}M_S(z),I_{\overline{R}(y)}=\vee_{z\in[y]R}I_S(z),N_{\overline{R}(y)}=\wedge_{z\in[y]R}N_S(z).$

Definition 2.6[5]: Let $(U,\tau_N(S))$ be a neutrosophic nano topological spaces, where $S\subseteq U$. Assume S and T be neutrosophic subset of U. Then the following hold:

1.$S\subseteq N_N cl(S)$.

2. S is neutrosophic nano closed iff $N_N cl(S)=S$.

3. $N_N cl(0_N)=0_N$ and $N_N cl(1_N)=1_N$.

4. $S\subseteq T$ Implies $N_N cl(S)\subseteq N_N cl(T)$.

5.$N_N cl(S\cup T)=N_N cl(S)\cup N_N cl(T)$.

6.$N_N cl(S\cap T)\subseteq N_N cl(S)\cap N_N cl(T)$.

7.$N_N cl(N_N cl(A))=N_N cl(S)$.

Definition 2.7[5]: Let $(U,\tau_N(S))$ be neutrosophic nano topological spaces, Where $S\subseteq U$. Assume S and T be neutrosophic subset of U. Then the following hold:

1.$N_N \operatorname{int}(S)\subseteq S$.

2. S is neutrosophic nano open iff $N_N \operatorname{int}(S)=S$.

3. $N_N \operatorname{int}(0_N)=0_N$ and $N_N \operatorname{int}(1_N)=1_N$.

4. $S\subseteq T$ implies $N_N \operatorname{int}(S)\subseteq N_N \operatorname{int}(T)$.

5.$N_N \operatorname{int}(S)\cup N_N \operatorname{int}(T)\subseteq N_N \operatorname{int}(S[T])$.

6.$N_N \operatorname{int}(S\cap T)=N_N \operatorname{int}(S)\cap N_N \operatorname{int}(T)$.

7.$N_N \operatorname{int}(N_N \operatorname{int}(A))=N_N \operatorname{int}(S)$.

Definition 2.8[5]: Let $(X,\tau_N(S))$ be a non-empty fixed set. A neutrosophic set A is an object having the form $S=\{\langle x,M_s(x),I_s(x),N_s(x)\rangle:x\in(X,\tau_N)\}$, where $M_s(x)$, $I_s(x),N_s(x)$ which represents the degree of membership, the degree of indeterminacy, and the degree of non-membership of each element $x\in(X,\tau_N)$ to the set S.

Definition 2.9[7]:

Let S and T be NS of the form $S=\{\langle x,M_s(x),I_s(x),N_s(x)\rangle:x\in(X,\tau_N)\}$ and $T=\{\langle x,M_T(x),I_T(x),N_T(x)\rangle:x\in(X,\tau_N)\}$.Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology ,we must introduce the NSS 0_N and 1_N as follows: 0_N may be defined as: $0_N = \{< x, 0, 0, 1 >: x \in X\}$

1_N may be defined as: $1_N = \{< x, 1, 1, 0 >: x \in X\}$

1. $S \subseteq T$ If and only if $M_S(x) \le M_T(x)$, $I_S(x) \le I_T(x)$ and $N_S(x) \ge N_T(x)$ for all $x \in (X, \tau_N)$
2. $S = T$ if and only if $S \subseteq T$ and $T \subseteq S$.
3. $S^C = \{< x, N_S(x), 1 - I_S(x), M_S(x) >: x \in (X, \tau_N)\}$.
4. $S \cup T = \{< x, M_S(x) \vee M_T(x), I_S(x) \vee I_T(x), N_S(x) \wedge N_T(x) >: x \in (X, \tau_N)\}$.
5. $S \cap T = \{< x, M_S(x) \wedge M_T(x), I_S(x) \wedge I_T(x), N_S(x) \vee N_T(x) >: x \in (X, \tau_N)\}$.

Definition 2.10[7]: Let $(X, \tau_N(S))$ be NTS and $S = \{\langle x, M_s(x), I_s(x), N_s(x) \rangle : x \in (X, \tau_N)\}$ be a as NS in X. Then the neutrosophic closure and neutrosophic interior of S defined by

1. $Ncl(S) = \bigcap\{K : K$ is an NCS in X and $S \subseteq K$ }.
2. $N\operatorname{int}(S) = \cup\{K : K$ is an NOS in X and $K \subseteq S\}$.

Definition 2.11[7]: Let $(U, \tau_N(S))$ be a neutrosophic nano topological space. Then a neutrosophic nano subset S in $(U, \tau_N(S))$ is said to be:

(a) Neutrosophic nano semi closed if $N_N \operatorname{int}(N_N cl(S)) \subseteq S$.

(b) Neutrosophic nano α -closed if $N_N cl(N_N \operatorname{int}(N_N cl(S))) \subseteq S$.

(c) Neutrosophic nano pre-closed if $N_N cl(N_N \operatorname{int}(S) \subseteq S$.

(d) Neutrosophic nano generalized-closed if $N_N cl(S) \subseteq V$ whenever $S \subseteq V$ and V is nano open in U.

(e) Neutrosophic nano semi generalized -closed if $N_N scl(S) \subseteq V$ whenever $S \subseteq V$ and V is nano semi open in U.

(f) Neutrosophic nano b-closed if $N_N \operatorname{int}(N_N cl(S)) \cap Ncl(N\operatorname{int}(S)) \subseteq S$

Definition 2.12: The difference between two fuzzy neutrosophic sets A and B is defined as

$A / B(X) = \{x, \min(T_A(x), F_B(x), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x)))\}$

Result 2.13:

Let A be neutrosophic nano gb-closed set $(U, \tau_N(S))$, then

$(1) Nbcl(S) = S \cup (N\operatorname{int}(Ncl(S) \cap Ncl(N\operatorname{int}(S)))$

$(2) N\operatorname{int}(S) = S \cap (Ncl(N\operatorname{int}(S) \cup N\operatorname{int}(Ncl(S)))$

3. NEUTROSOPHIC NANO gb –CLOSED SETS

Definition 3.1: A subset S of a neutrosophic nano topological space $(U, \tau_N(S))$ is called neutrosophic nano generalized closed (briefly, neutrosophic nano gb-closed), if $N_N bcl(S) \subseteq V$ whenever $S \subseteq V$ and V is neutrosophic nano open in U.

Theorem 3.2:

(i) Every neutrosophic nano closed set is neutrosophic nano-gb closed set
(ii) Every neutrosophic nano b-closed set is neutrosophic nano-gb closed set
(iii) Every neutrosophic nano g-closed set is neutrosophic nano-gb closed set.
(iv) Every neutrosophic nano α -closed set is neutrosophic nano-gb closed set.
(v) Every neutrosophic nano sg-closed set is neutrosophic nano-gb closed set.
(vi) Every neutrosophic nano semi-closed set is neutrosophic nano-gb closed set.

Proof: Let S be neutrosophic nano closed set $N_N cl(S) = S$. Assume that $S \subseteq V$ and V is neutrosophic nano open in U. Since $N_N bcl(S) \subseteq N_N cl(S)$. Then $N_N bcl(S) \subseteq V$, whenever $S \subseteq V$ and V is neutrosophic nano open in U. Therefore S is neutrosophic nano generalized b-closed set. Proof is obvious for others.

Remark 3.3: The Converse of the above theorem need not be true which can be seen from the following examples.

Example 3.4: Assume $U = \{n_1, n_2, n_3\}$ be the Universe set and the equivalence relation is $U/R = \{\{n_1, n_3\}, \{n_2\}\}$. Let $S = \left\{\left\langle \frac{n_1}{(0.1, 0.2, 0.3)} \right\rangle, \left\langle \frac{n_2}{(0.2, 0.3, 0.4)} \right\rangle, \left\langle \frac{n_3}{(0.1, 0.6, 0.4)} \right\rangle\right\}$ be neutrosophic nano subset of U. $\underline{N}(S) = \left\{\left\langle \frac{n_1, n_3}{(0.1, 0.2, 0.4)} \right\rangle, \left\langle \frac{n_2}{(0.2, 0.3, 0.4)} \right\rangle\right\}$ and $\bar{N}(S) = \left\{\left\langle \frac{n_1, n_3}{(0.1, 0.6, 0.3)} \right\rangle, \left\langle \frac{n_2}{(0.2, 0.3, 0.4)} \right\rangle\right\}$ $B_N(S) = \left\{\left\langle \frac{n_1, n_3}{(0.1, 0.6, 0.4)} \right\rangle, \left\langle \frac{n_2}{(0.2, 0.3, 0.4)} \right\rangle\right\}$ .Here $\tau_N(S) = \{0_N, 1_N, \underline{N}(S), \bar{N}(S), B_N(S)\}$ be a neutrosophic nano open set and a neutrosophic nano closed set is $\tau_N{}^c(S) = \{0_N, 1_N, \underline{N}^c(S), \bar{N}^c(S), B_N(S)\}$. where

$$\underline{N}^c(S) = \left\{\left\langle \frac{n_1, n_3}{(0.4, 0.8, 0.1)} \right\rangle, \left\langle \frac{n_2}{(0.4, 0.7, 0.2)} \right\rangle\right\}$$

$\bar{N}^c(S) = \left\{\left\langle \frac{n_1, n_3}{(0.3, 0.4, 0.1)} \right\rangle, \left\langle \frac{n_2}{(0.4, 0.7, 0.2)} \right\rangle\right\}$ and $B_N{}^c(S) = \left\{\left\langle \frac{n_1}{(0.4, 0.4, 0.1)} \right\rangle, \left\langle \frac{n_2, n_3}{(0.4, 0.7, 0.2)} \right\rangle\right\}$

.Assume $R = \left\{\left\langle \frac{n_1}{(0.2, 0.1, 0.3)} \right\rangle, \left\langle \frac{n_2}{(0.3, 0.1, 0.2)} \right\rangle, \left\langle \frac{n_3}{(0.1, 0.2, 0.3)} \right\rangle\right\}$ be a neutrosophic nano gb-closed but it is not neutrosophic nano closed set.

Example 3.5:

Assume $U = \{n_1, n_2, n_3\}$ be the Universe set and the equivalence relation is $U/R = \{\{n_1\}, \{n_2, n_3\}\}$. Let $S = \left\{\left\langle \frac{n_1}{0.1, 0.2, 0.3} \right\rangle, \left\langle \frac{n_2}{0.3, 0.2, 0.1} \right\rangle, \left\langle \frac{n_3}{0.1, 0.2, 0.2} \right\rangle\right\}$ be neutrosophic nano subset of U

$\underline{N}(S)=\left\{\left\langle \frac{n_1}{0.1,0.2,0.3}\right\rangle ,\left\langle \frac{n_2,n_3}{0.1,0.2,0.2}\right\rangle \right\}$ and $\bar{N}(S)=\left\{\left\langle \frac{n_1}{0.1,0.2,0.3}\right\rangle ,\left\langle \frac{n_2,n_3}{0.3,0.2,0.1}\right\rangle \right\}$

$B_N(S)=\left\{\left\langle \frac{n_1}{0.1,0.2,0.3}\right\rangle ,\left\langle \frac{n_2,n_3}{0.1,0.2,0.2}\right\rangle \right\}$.Here $\tau_N(S)=\{0_N,1_N,\underline{N}(S),\bar{N}(S),B_N(S)\}$ be a neutrosophic nano open set and a neutrosophic nano closed set is $\tau_N{}^c(S)=\{0_N,1_N,\underline{N}^c(S),\bar{N}^c(S),B_N(S)\}$. where $\underline{N}^c(S)=\left\{\left\langle \frac{n_1}{0.3,0.8,0.1}\right\rangle ,\left\langle \frac{n_2,n_3}{0.2,0.8,0.1}\right\rangle \right\}$

$\bar{N}^c(S)=\left\{\left\langle \frac{n_1}{0.3,0.8,0.1}\right\rangle ,\left\langle \frac{n_2,n_3}{0.1,0.8,0.3}\right\rangle \right\}$ and $B_N{}^c(S)=\left\{\left\langle \frac{n_1}{0.3,0.8,0.1}\right\rangle ,\left\langle \frac{n_2,n_3}{0.2,0.8,0.1}\right\rangle \right\}$. Assume $R=\left\{\left\langle \frac{n_1}{0.2,0.1,0.3}\right\rangle ,\left\langle \frac{n_2}{0.3,0.1,0.2}\right\rangle ,\left\langle \frac{n_3}{0.1,0.2,0.3}\right\rangle \right\}$ be a neutrosophic nano gb-closed but it is not neutrosophic nano b-closed set.

Example 3.6: Assume $U=\{n_1,n_2,n_3,n_4,n_5\}$ be the Universe set and the equivalence relation is $U/R=\{\{n_1,n_3\},\{n_2\},\{n_4,n_5\}\}$. Let

$S=\left\{\left\langle \frac{n_1}{(0.4,0.3,0.4)}\right\rangle ,\left\langle \frac{n_2}{(0.5,0.3,0.5)}\right\rangle ,\left\langle \frac{n_3}{(0.5,0.3,0.2)}\right\rangle ,\left\langle \frac{n_4}{(0.6,0.3,0.1)}\right\rangle ,\left\langle \frac{n_5}{(0.5,0.3,0.1)}\right\rangle \right\}$ be neutrosophic nano subset of U. $\underline{N}(S)=\left\{\left\langle \frac{n_1,n_3}{(0.4,0.3,0.4)}\right\rangle ,\left\langle \frac{n_2}{(0.5,0.3,0.5)}\right\rangle ,\left\langle \frac{n_4,n_5}{(0.5,0.3,0.1)}\right\rangle \right\}$ and

$\bar{N}(S)=\left\{\left\langle \frac{n_1,n_3}{(0.5,0.3,0.2)}\right\rangle ,\left\langle \frac{n_2}{(0.5,0.3,0.5)}\right\rangle ,\left\langle \frac{n_4,n_5}{(0.6,0.3,0.1)}\right\rangle \right\}$

$B_N(S)=\left\{\left\langle \frac{n_1,n_3}{(0.4,0.3,0.4)}\right\rangle ,\left\langle \frac{n_2}{(0.5,0.3,0.5)}\right\rangle ,\left\langle \frac{n_4,n_5}{(0.5,0.3,0.1)}\right\rangle \right\}$.Here

$\tau_N(S)=\{0_N,1_N,\underline{N}(S),\bar{N}(S),B_N(S)\}$ be a neutrosophic nano open set and a neutrosophic nano closed set is $\tau_N{}^c(S)=\{0_N,1_N,\underline{N}^c(S),\bar{N}^c(S),B_N(S)\}$. where

$\underline{N}^c(S)=\left\{\left\langle \frac{n_1,n_3}{(0.4,0.7,0.4)}\right\rangle ,\left\langle \frac{n_2}{(0.5,0.7,0.5)}\right\rangle ,\left\langle \frac{n_4,n_5}{(0.1,0.7,0.5)}\right\rangle \right\}$

$\bar{N}^c(S)=\left\{\left\langle \frac{n_1,n_3}{(0.2,0.7,0.5)}\right\rangle ,\left\langle \frac{n_2}{(0.5,0.7,0.5)}\right\rangle ,\left\langle \frac{n_4,n_5}{(0.1,0.7,0.6)}\right\rangle \right\}$ and

$B_N{}^c(S)=\left\{\left\langle \frac{n_1,n_3}{(0.4,0.7,0.4)}\right\rangle ,\left\langle \frac{n_2}{(0.5,0.7,0.5)}\right\rangle ,\left\langle \frac{n_4,n_5}{(0.1,0.7,0.5)}\right\rangle \right\}$.Assume

$R=\left\{\left\langle\frac{n_1}{(0.6,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.5,0.3,0.3)}\right\rangle,\left\langle\frac{n_3}{(0.5,0.2,0.3)}\right\rangle,\left\langle\frac{n_4}{(0.3,0.3,0.1)}\right\rangle,\left\langle\frac{n_5}{(0.4,0.4,0.1)}\right\rangle\right\}$ be a neutrosophic nano gb-closed but it is not neutrosophic nano g-closed set.

Example 3.7: Assume $U=\{n_1,n_2,n_3\}$ be the Universe set and the equivalence relation is $U/R=\{\{n_1\},\{n_2,n_3\}\}$. Let $S=\left\{\left\langle\frac{n_1}{(0.3,0.4,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.6,0.3,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.6,0.2)}\right\rangle\right\}$ be neutrosophic nano subset of U. $\underline{N}(S)=\left\{\left\langle\frac{n_1}{(0.3,0.4,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.2,0.3,0.2)}\right\rangle\right\}$ and $\bar{N}(S)=\left\{\left\langle\frac{n_1}{(0.3,0.4,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.6,0.6,0.1)}\right\rangle\right\}$ $B_N(S)=\left\{\left\langle\frac{n_1}{(0.3,0.4,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.2,0.6,0.2)}\right\rangle\right\}$.Here $\tau_N(S)=\{0_N,1_N,\underline{N}(S),\bar{N}(S),B_N(S)\}$ be a neutrosophic nano open set and a neutrosophic nano closed set is $\tau_N{}^c(S)=\{0_N,1_N,\underline{N}^c(S),\bar{N}^c(S),B_N(S)\}$. where $\underline{N}^c(S)=\left\{\left\langle\frac{n_1}{(0.3,0.6,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.2,0.7,0.2)}\right\rangle\right\}$

$\bar{N}^c(S)=\left\{\left\langle\frac{n_1}{(0.3,0.6,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.1,0.4,0.6)}\right\rangle\right\}$ and $B_N{}^c(S)=\left\{\left\langle\frac{n_1}{(0.3,0.6,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.2,0.4,0.2)}\right\rangle\right\}$

.Assume $R=\left\{\left\langle\frac{n_1}{(0.2,0.1,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.3,0.1,0.2)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.1,0.2)}\right\rangle\right\}$ be a neutrosophic nano gb-closed but it is not neutrosophic nano α - closed set.

Example 3.8: Assume $U=\{n_1,n_2,n_3\}$ be the Universe set and the equivalence relation is $U/R=\{\{n_1\},\{n_2,n_3\}\}$. Let $S=\left\{\left\langle\frac{n_1}{(0.1,0.2,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.3,0.4,0.5)}\right\rangle,\left\langle\frac{n_3}{(0.6,0.4,0.1)}\right\rangle\right\}$ be neutrosophic nano subset of U. $\underline{N}(S)=\left\{\left\langle\frac{n_1}{(0.1,0.2,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.3,0.4,0.5)}\right\rangle\right\}$ and $\bar{N}(S)=\left\{\left\langle\frac{n_1}{(0.1,0.2,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.6,0.4,0.1)}\right\rangle\right\}$, $B_N(S)=\left\{\left\langle\frac{n_1}{(0.1,0.2,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.3,0.4,0.5)}\right\rangle\right\}$.Here $\tau_N(S)=\{0_N,1_N,\underline{N}(S),\bar{N}(S),B_N(S)\}$ be a neutrosophic nano open set and a neutrosophic nano closed set is $\tau_N{}^c(S)=\{0_N, 1_N, \underline{N}^c(S), \bar{N}^c(S), B_N(S)\}$. where

$\underline{N}^c(S)=\left\{\left\langle\frac{n_1}{(0.3,0.8,0.1)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.5,0.6,0.3)}\right\rangle\right\}$

$\bar{N}^c(S)=\left\{\left\langle\frac{n_1}{(0.3,0.8,0.1)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.1,0.6,0.6)}\right\rangle\right\}$ and $B_N{}^c(S)=\left\{\left\langle\frac{n_1}{(0.3,0.8,0.1)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.5,0.6,0.3)}\right\rangle\right\}$

.Assume $R=\left\{\left\langle\frac{n_1}{(0.1,0.2,0.4)}\right\rangle,\left\langle\frac{n_2}{(0.5,0.4,0.3)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.3)}\right\rangle\right\}$ be a neutrosophic nano gb-closed but it is not neutrosophic nano sg -closed set.

Example 3.9: Assume $U=\{n_1,n_2,n_3\}$ be the Universe set and the equivalence relation is $U/R=\{\{n_1\},\{n_2,n_3\}\}$. Let $S=\left\{\left\langle\frac{n_1}{(0.1,0.2,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.3,0.2,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.1,0.2,0.2)}\right\rangle\right\}$ be neutrosophic nano subset of U. $\underline{N}(S)=\left\{\left\langle\frac{n_1}{(0.1,0.2,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.1,0.2,0.2)}\right\rangle\right\}$ and

$\bar{N}(S)=\left\{\left\langle\frac{n_1}{(0.1,0.2,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.3,0.2,0.1)}\right\rangle\right\}$ $B_N(S)=\left\{\left\langle\frac{n_1}{(0.1,0.2,0.3)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.1,0.2,0.2)}\right\rangle\right\}$.Here

$\tau_N(S)=\{0_N,1_N,\underline{N}(S),\bar{N}(S),B_N(S)\}$ be a neutrosophic nano open set and a neutrosophic nano closed set is $\tau_N{}^c(S)=\{0_N,1_N,\underline{N}^c(S),\bar{N}^c(S),B_N(S)\}$. where

$\underline{N}^c(S)=\left\{\left\langle\frac{n_1}{(0.3,0.8,0.1)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.2,0.8,0.1)}\right\rangle\right\}$

$\bar{N}^c(S)=\left\{\left\langle\frac{n_1}{(0.3,0.8,0.1)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.1,0.8,0.3)}\right\rangle\right\}$ and $B_N{}^c(S)=\left\{\left\langle\frac{n_1}{(0.3,0.8,0.1)}\right\rangle,\left\langle\frac{n_2,n_3}{(0.2,0.8,0.1)}\right\rangle\right\}$

Assume $R=\left\{\left\langle\frac{n_1}{(0.2,0.1,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.3,0.1,0.2)}\right\rangle,\left\langle\frac{n_3}{(0.1,0.2,0.3)}\right\rangle\right\}$ be a neutrosophic nano gb-closed but it is not neutrosophic nano semi closed set.

Theorem 3.10: The union of two neutrosophic nano gb-closed set is also neutrosophic nano gb-closed set.

Proof: Let us assume that S and T be two neutrosophic nano gb-closed set. Let $S\cup T\subseteq V$, V is neutrosophic nano open in U. By definition of neutrosophic nano gb-closed set $N_N bcl(S)\subseteq V$ and $N_N bcl(T)\subseteq V$ .This implies that $N_N bcl(S\cup T)\subseteq V$ .Hence $S\cup T$ is neutrosophic nano gb-closed set.

Theorem 3.11: Let S be neutrosophic nano open and neutrosophic nano gb-closed set. Then, $S \cap T$ is neutrosophic nano gb-closed whenever $T \in N_N bcl(U, X)$.

Proof: Let S be neutrosophic nano open and neutrosophic nano gb-closed, then $N_N bcl(S) \subseteq S$ and $S \subseteq N_N bcl(S)$.Thus S is neutrosophic nano b-closed .Hence $S \cap T$ is neutrosophic nano b-closed in U. We know that every neutrosophic nano b-closed set is neutrosophic nano gb-closed [Theorem 3.2] which implies that $S \cap T$ is neutrosophic nano gb-closed in U.

Theorem 3.12: Assume S be a neutrosophic nano gb-closed set in neutrosophic nano topological spaces U. Then $N_N bcl(S) - S$ does not contain any non-empty neutrosophic nano closed set.

Proof: Let S be neutrosophic nano gb-closed in U, and T be neutrosophic nano closed subset of $N_N bcl(S) - S$ That is $T \subseteq N_N bcl(S) - S$ which implies that $T \subseteq N_N bcl(S) \cap S^c$.That is $T \subseteq N_N bcl(S)$ and $T \subseteq S^c$ implies that $S \subseteq T^c$ where T^c is a neutrosophic nano open set. Since S is neutrosophic nano gb-closed $N_N bcl(S) \subseteq T^c$.That is $T \subseteq [N_N bcl(S)]^c$.Thus $T \subseteq N_N bcl(S) \cap [N_N bcl(S)]^c = \phi$

Therefore $T = \phi$.

Theorem 3.13: If S is neutrosophic nano gb-closed set and $S \subseteq T \subseteq N_N bcl(S)$ then T is neutrosophic nano gb-closed set.

Proof: Let $T \subseteq U$, Where U is neutrosophic nano open set in neutrosophic nano topological spaces. Then $S \subseteq T$ implies $S \subseteq U$ .Since S is neutrosophic nano gb-closedset $N_N bcl(S) \subseteq U$. Also $T \subseteq N_N bcl(S)$ implies $N_N bcl(T) \subseteq N_N bcl(S)$.This shows that $N_N bcl(S) \subseteq U$ and so T is neutrosophic nano gb-closed set.

NEUTROSOPHIC NANO gb-OPEN SETS

Definition 3.14: A subset S of a neutrosophic nano topological space $(U, \tau_N(S))$ is called neutrosophic nano gb-open set if S^c is neutrosophic nano gb-closed.

Theorem 3.15: A subset $S \subseteq U$ is neutrosophic nano gb-open if and only if $T \subseteq N_N b\text{int}(S)$ whenever T is neutrosophic nano closed set and $T \subseteq S$.

Proof: Let S be neutrosophic nano gb-open set and suppose $T \subseteq S$ where T is neutrosophic nano closed. Then $U - S$ is neutrosophic nano gb-closed set contained in the neutrosophic nano open set $U - T$ .Hence $N_N bcl(U - S) \subseteq U - T$ and $U - N_N b\text{int}(S) \subseteq U - T$. Thus $T \subseteq N_N b\text{int}(S)$.

Conversely, if T is a neutrosophic nano closed set with $T \subseteq N_N b\text{int}(S)$ and $T \subseteq S$, then $U - N_N b\text{int}(S) \subseteq U - T$. Thus $N_N bcl(U - S) \subseteq U - T$. Hence $U - S$ is a neutrosophic nano gb-closed set and S is a neutrosophic nano gb-open set.

Theorem 3.16: If S is neutrosophic nano gb-open, V is neutrosophic nano open and $N_N b\text{int}(S) \cup S^c \subseteq V$ then $V = U$.

Proof: Let S be neutrosophic nano gb-open and V is neutrosophic nano open such that $N_N b\text{int}(S) \cup S^c \subseteq V$. Then $V^c \subseteq S \cap N_N bcl(S^c) \subseteq N_N bcl(S^c) - S^c$. Since S^c is neutrosophic nano gb-closed, $N_N bcl(S^c) - S^c$ cannot contain any non-empty neutrosophic nano closed set .But V^c is a neutrosophic nano closed subset of $N_N bcl(S^c) - S^c$.Therefore $V^c = \phi$.That is V=U.

Theorem 3.17: If $N_N b\text{int}(S) \subseteq T \subseteq S$ and if S is neutrosophic nano gb-open, then T is neutrosophic nano gb-open.

Proof: Let $N_N b\text{int}(S) \subseteq T \subseteq S$, then $S^c \subseteq T^c \subseteq N_N bcl(S^c)$, where S^c is neutrosophic nano gb-closed set and hence T^c is also neutrosophic nano gb-closed by Theorem 3.15. Therefore, T is neutrosophic nano gb-open.

Theorem 3.18: If S is neutrosophic nano gb-closed then $N_N bcl(S) - S$ is neutrosophic nano gb-open.

Proof: Let S be neutrosophic nano gb-closed. Let T be neutrosophic nano closed set such that $T \subseteq N_N bcl(S) - S$. Then $T = \phi$.Since $N_N bcl(S) - S$ cannot contain any non-empty neutrosophic nano closed set. Therefore $T \subseteq N_N b\text{int}(N_N bcl(S) - S)$ .Hence $N_N bcl(S) - S$ is neutrosophic nano gb-open.

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