A CHROMATIC CORE SUBGRAPH OF PROUCT OF GRAPH WHICH ADMITS JOHAN COLORING

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Abstract

A vertex v of a given graph is said to be in a rainbow neighbourhood of G if every color class of G consists of at least one vertex from the closed neighbourhood N[v]. A maximal proper coloring of a graph G is a Johan coloring if and only if every vertex of G belongs to a rainbow neighbourhood of G. In general, all graphs need not have a Johan coloring, even though they admit a chromatic coloring. In this paper, we have found Chromatic core subgraph of Product of graph such as Cartesian product and Lexicographical product of graphs of path, cycles and complete graphs which admit Johan coloring. Ideas for further research are given.

Keywords: Cartesian Product, Chromatic Core Subgraph, Johan Coloring, Johan Number, Lexicographical product, Rainbow neighbourhood.

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I INTRODUCTION

For general notation and concepts in graphs and digraphs see [1, 3, 7]. We will write that a graph G has order v (G) = $n \geq 1$ and size ε(G) = $p \geq 0$ with minimum and maximum degree δ (G) and Δ(G), respectively. Unless mentioned otherwise, all graphs G are finite, undirected simple graphs. We recall that if C = {c_1, c_2 . . . c_l} and l sufficiently large, is a set of distinct colors, a proper vertex coloring of a graph G denoted $\varphi : V(G) \mapsto C$ is a vertex coloring such that no two distinct adjacent vertices have the same color. The cardinality of a minimum set of colors which allows a proper vertex coloring of G is called the chromatic number of G and is denoted $\chi(G)$. In this paper extended to the that paper, here we extended the product as Cartesian product and Lexicographical product of Path, cycle and complete graphs.

II CHROMATIC CORE SUBGRAPH AND JOHAN COLORING

For a graph G its structural size is measured by its structure index denoted and defined as, si (G) = v (G) + ε(G). We say that the smaller of graphs G and H is the graph satisfying the condition, min {si (G), si (H)}. If si (G) = si (H) the graphs are of equal structural size but not necessarily isomorphic. A straight forward example is the path, P_4 and the star graph, S_3.
The notion of a rainbow neighbourhood of a graph G with a chromatic coloring C has been defined in [6] as the closed neighbourhood N[v] of a vertex $v \in V$ (G) which contains at least one colored vertex of each color in the chromatic coloring C of G. Motivated by this study, a new graph coloring, namely Johan coloring is admitted to graphs as follows.

For a finite, undirected simple graph G of order v (G) = n ≥ 1 a Chromatic core subgraph H is a smallest induced subgraph H (smallest in respect of si (H)) such that, $\chi(H) = \chi(G)$.

A proper k-coloring C of a graph G is called the Johan coloring or the J-coloring of G if C is the maximal coloring such that every vertex of G belongs to a rainbow neighbourhood of G. A graph G is J-colorable if it admits a J-coloring. The J-coloring number of a graph G, denoted by J (G), is the maximum number of colors in a J-coloring of G.

III CHROMATIC CORE SUBGRAPH ADMITS JOHAN COLORING OF CARTESIAN PRODUCT OF GRAPHS

In respect of the cartesian G□H the following results holds. Recall that in the Cartesian product of G□H, the nodes (v_i,u_j) and (v_n,u_m) are adjacent if and only if, $v_i = v_n$ and u_j adj u_m or $u_j = u_m$ and v_i adj v_n.

Theorem 3.1 Let G and H be any two graphs then the J-CCS of G□H is K_2, for G and H can be path or an even cycle or K_n (n = 1).

Proof: Let G and H be any two graphs which can be a path or an even cycle. Let V (G)= { $u_1,u_2, \cdots, u_m$ } and V (H)= { $v_1,v_2, \cdots, v_n$ }. Now, V(G□H) = V (G)×V (H) then draw an edge between (u_i,v_j) and (u_m,v_n) if the following conditions are satisfied, (i.e) if $u_i = u_m$ and v_j adj v_n or $v_j = v_n$ and u_i adj u_m. The Cartesian product graph G□H admits Johan coloring and its J-number is 2. Also, there exists a subgraph K_2 with J-number as 2. Hence, the J-CCS of G□H is K_2.

Theorem 3.2 Let K_m and K_n be complete graphs, then

$$J-CCS(K_m \square K_n) = \begin{cases} K_m & if \quad n \leq m-1 \\ K_n & if \quad n > m-1 \end{cases}$$

Proof: Let K_m and K_n be both complete graphs.

Case (i): $n \leq m-1$ Let $V(K_m) = \{u_1,u_2, \cdots, u_m\}$ and $V(K_n)=\{v_1,v_2, \cdots, v_n\}$. Now, $V(K_m \square K_n)$= V (K_m)×V (K_n), then draw a link between (u_i,v_j) and (u_m,v_n) if the following conditions are satisfied, (i.e) if $u_i = u_m$ and v_j adj v_n or $v_j = v_n$ and u_i adj u_m. The Cartesian product graph $K_m \square K_n$ admits Johan coloring and its J-number is m. Also, there exists a subgraph K_m with J-number as m. Hence, the J-CCS of $K_m \square K_n$ is K_m, for any complete graph of $n \leq m-1$.

Case (ii): $n > m-1$ The proof is similar, but here the J-number is n. Also, there exists a subgraph K_n with J-number as n. Hence, the J-CCS of $K_m \square K_n$ is K_n.

Theorem 3.3 Let C_m and C_3 be cycle graphs, then the J-CCS of $C_m \square C_3$ is K_3.

Proof: Let C_m and C_3 be any two cycle graphs. Let $V(C_m) = \{u_1,u_2,...,u_m\}$ and $V(C_3) = \{v_1,v_2,v_3\}$. Now, $V(C_m \square C_3)=V(C_m) \times V(C_3)$ then draw a link between (u_i,v_j) and (u_m,v_3) if the following conditions are satisfied, (i.e) if $u_i = u_m$ and v_j adj v_3 or $v_j = v_3$ and u_i adj u_m. The Cartesian product graph $C_m \square C_3$ admits Johan coloring and its J-number is 3. Also, there exists a subgraph K_3 with J-number as 3. Hence, the J-CCS of $C_m \square C_3$ is K_3.

Example 3.4

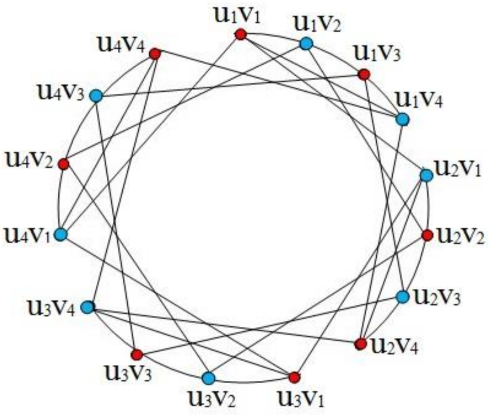

Figure 1: Cartesian product of $C_4 \square C_4$

Theorem 3.5 Let G be path P_m or an even cycle graph C_m and K_n be a complete graph with n nodes, n > 1. The J-CCS of $G\square K_n$ is K_n.

Proof: Let G be a path or an even cycle graph and K_n be a complete graph with n > 1. Let $V(G) = \{u_1,u_2,\cdots ,u_m\}$ and $V(K_n) = \{v_1,v_2,\cdots ,v_n\}$. Now $V(G\square K_n)= V(G) \times V(K_n)$, then draw a link between (u_i,v_j) and (u_m,v_n) if the following conditions are satisfied, (i.e) if $u_i = u_m$ and v_j adj v_n or $v_j = v_n$ and u_i adj u_m. The Cartesian product graph $G\square K_n$ admits Johan coloring and its J-number is n. Also, there exists a subgraph K_n with J-number as n. Hence, the J-CCS of $G\square K_n$ is K_n.

Corollary 3.6 The J-CCS of $K_n\square G$ is K_n if n > 1, for G can be a path P_m or even cycle C_m.

IV J-CHROMATIC CORE SUBGRAPH OF LEXICOGRAPHICAL PRODUCT OF GRAPHS

The section begins with the result for Lexicographical product of graphs $G \otimes H$. Recall that in the lexicographical product $G \otimes H$, the nodes (u_i,v_j) and (u_m,v_n) are adjacent if and only if, $u_i = u_m$ and v_j adj v_n or u_i adj u_m.

Theorem 4.1: The J-CCS of $P_m \otimes P_n$ is K_4.

Proof: Let P_m and P_n be path graphs. The node set of P_n and P_m are given by, $V(P_m) = \{u_1,u_2,\cdots ,u_m\}$ and $V(P_n) = \{v_1,v_2,\cdots ,v_n\}$. The nodes (u_i,v_j) and (u_m,v_n) are adjacent if, $u_i = u_m$ and v_j adj v_n or u_i adj u_m. The lexicographical product graphs $P_m \otimes P_n$ admits a Johan coloring and its J-number is 4. Also, there exists a subgraph K_4 with J-number 4. Hence, the J-CCS of $P_n \otimes P_m$ is K_4.

Theorem 4.3: If K_m and K_n are two complete graphs, the J-CCS of $K_m \otimes K_n$ is K_{mn}.

Proof: Let K_m and K_n be both complete graphs. The node set of K_m and K_n are given by, $V(K_m) = \{u_1,u_2,\cdots ,u_m\}$ and $V(K_n) = \{v_1,v_2,\cdots ,v_n\}$. The nodes (u_i,v_j) and (u_m,v_n) are adjacent if, $u_i = u_m$ and v_j adj v_n or u_i adj u_m. The Lexicographical product graph $K_m \otimes K_n$

admits Johan coloring and its J-number is mn. Also, there exists a subgraph K_{mn} with J-number as mn. Hence, the J-CCS of $K_m \otimes K_n$ is K_{mn}.

Example 4.2

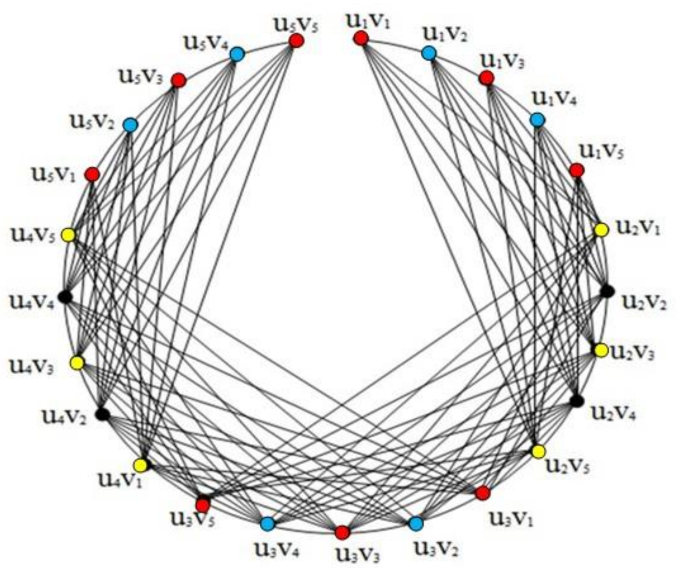

Figure 2: Lexico Graphical Product of Graphs $P_5 \otimes P_5$

Theorem 4.4: If P_m is a path of m nodes and K_n is a complete graph of n nodes, the J-CCS of $P_m \otimes K_n$ is K_{2n}

Proof: The proof is similar to that of theorem 4.3. Since, the J-CCS of P_m is P_2. $J(P_m \otimes K_n)=2n$, which implies that the J-CCS of $(P_m \otimes K_n) = K_{2n}$.

Theorem 4.5: If G is a path graph P_m or an even cycle C_m and C_2 is a Cycle graph with n nodes, the J-CCS of $G \otimes C_n$ is given as follows:

$$J-CCS(G\otimes C_n) = \begin{cases} K_4 & \text{if } n \text{ is even} \\ K_6 & \text{if } n \text{ is odd} \end{cases}$$

Proof: Case (i) When n is even Let G be a path or an even cycle with m nodes, and C_n is cycle with n nodes. Let $V(G) = \{u_1,u_2,\cdots,u_m\}$ and $V(C_n) = \{v_1,v_2,\cdots,v_n\}$. The nodes (u_i,v_j) and (u_m,v_n) are adjacent if, $u_i = u_m$ and v_j adj v_n or u_i adj u_m. The lexicographical product graph $G\otimes C_n$ admits Johan coloring and its J-number is 4. Also, there exists a subgraph K_4 with J-number as 4. Hence, The J-CCS of $G \otimes C_n$ is K_4.

Case (ii) For Odd n The proof is similar.

Theorem 4.6:

$$J-CCS(C_m \otimes C_n) = \begin{cases} K_4 & \text{if } n \text{ is even, m is odd} \\ K_9 & \text{if } n \text{ is odd, m is even} \end{cases}$$

Proof: The proof is similar to that of theorem 4.5.

Theorem 4.7: If K_n is a Complete graph with n nodes and C_m is a Cycle graph with m nodes, the J-CCS of $K_n \otimes C_m$ is given as follows:

$$J-CCS(K_n \otimes C_m) = \begin{cases} C_m & \text{if } n=1 \\ K_{2n} & \text{if } n \text{ is even, } n\geq 2 \\ K_{3n} & \text{if } n \text{ is odd, } n\geq 2 \end{cases}$$

Proof: Case (i) For n = 1, the proof is trival.
Case (ii) $n \geq 2$, n is even. Let K_n and C_m be a complete graph and an even cycle respectively. Let $V(K_n) = \{u_1,u_2,\cdots,u_n\}$ and $V(C_m) = \{v_1,v_2,\cdots,v_m\}$. The nodes (u_i,v_j) and (u_n,v_m) are adjacent if, $u_i = u_n$ and v_j adj v_m or u_i adj u_n. To color the nodes of $K_n \otimes C_m$, it admits Johan coloring and its J-number is 2n. Also, there exists a subgraph K_{2n} with J-number 2n. Hence, the J-CCS of $K_n \otimes C_m$ is K_{2n}.

Case (iii) $n \geq 3$, n is even. Let $V(K_n) = \{u_1,u_2,\cdots,u_n\}$ and $V(C_m) = \{v_1,v_2,\cdots,v_m\}$. The nodes (u_i,v_j) and (u_n,v_m) are adjacent if, $u_i = u_n$ and v_j adj v_m or u_i adj u_n. On J - coloring the nodes of $K_n \otimes C_m$, we obtain its J-number as 3n. Also, there exists a subgraph K_{3n} with J-number is 3n. The J-CCS of $K_n \otimes C_m$ is K_{3n}.

V CONCLUSION

In this paper, The Johan coloring of product of graphs namely, Cartesian product and Lexicographical product of path, cycle and complete graphs were studied and their Johan Chromatic Core Subgraph were obtained.

Further, The field of research can be developed by obtaining the notion of J Chromatic Core Subgraph of graphs, other colorings such as edge coloring, local coloring, dynamic coloring, co-coloring, Grundy coloring etc., which are still open**.**

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