AN INTUTIONISTIC FUZZY GEOMETRIC PROGRAMMING APPROACH FOR AN EOQ MODEL WITH ADVANTAGES AND BENEFITS OF BIO-PACKAGING AND ECO-LABELING OF THE PRODUCT

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ABSTRACT

In this paper an Economic order quantity model with package cost and focuses on the need of packaging , labeling of the products along with the advantages and benefits of bio-packaging and eco-labeling which speed up the product promotion in intutionistic fuzzy environment. We propose intutionistic fuzzy geometric programming method for minimizing the total cost .

KEYWORDS: Economic order quantity, Geometric programming , Intutionistic fuzzy geometric programming method, Bio-Packaging and Eco-Labeling cost.

1. INTRODUCTION

Geometric Programming (GP) is an effective method to solve a non-linear programming problem. It has certain advantages over the other optimization methods. Here, the advantages are that is usually much simpler to work with the dual than primal. Degree of Difficulty plays a significant role for solving a non-linear programming problem by GP method. Since late 1960, GP has been known and used in various fields (like OR, Engineering Sciences etc.). Duffin, Petersen and Zener (1966) discussed the basic theories with engineering applications in their books.

Another famous book on GP and its application appeared in Beightler and Philips (1976). There are many references on application and the methods of GP in the survey papers (like Eckar (1980), Beightler et.al. (1979), Zener (1971). Hariri et. al. (1997) discussed the multi-item production lot-size inventory model with varying order cost under a restriction Jung and Klain (2001) developed single item inventory problems and solved by GP method. Ata Fragany and Wakeel (2003) considered some inventory problems solved by GP technique. Zadeh (1965) first gave the concept of fuzzy set theory.

Roy and Maiti (1997) solved single objective fuzzy EOQ model by GP technique. Recently Mondal et. al. (2005) developed a multi-objective inventory model and solved it by GP method. A multiobjective fuzzy economic production quantity model is solved using GP approach by Islam and Roy (2004).

Intuitionistic Fuzzy Set (IFS) was introduced by K. Atanassov (1986) and seems to be applicable to real world problems. The concept of IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. Thus it is expected that, IFS can be used to simulate human decision-making process and any activitities requiring human expertise and knowledge that are inevitably imprecise or totally reliable. Here the degree of rejection and satisfaction are considered so that the sum of both values is always less than unity (1986). Atanossov also analyzed Intuitionistic fuzzy sets in a more explicit way. Atanassov(1989) discussed an Open problems in intuitionistic fuzzy sets theory.

An Interval valued intuitionistic fuzzy sets was analyzed by Atanassov and Gargov(1999). Atanassov and Kreinovich(1999) implemented Intuitionistic fuzzy interpretation of interval data. Banerjee and Roy (2009) considered application of the Intuitionistic Fuzzy Optimization in the Constrained . A Constrained Stochastic Inventory Model 191 Multi-Objective Stochastic Inventory Model. Banerjee and Roy (2010) also discussed the solution of Single and Multi-Objective Stochastic Inventory Models with Fuzzy Cost Components by Intuitionistic Fuzzy Optimization Technique.

In this paper an Economic order quantity model with package cost and focuses on the need of packaging , labeling of the products along with the advantages and benefits of bio-packaging and eco-labeling which speed up the product promotion in intutionistic fuzzy environment.

2. PRELIMINARIES

2.1 Intutionistic Fuzzy set:

Let X is a non-empty set, An Intutionisitic fuzzy set $\overline{A} = \{ < x, \mu_A(x) , v_A(x) > / x \epsilon X \}$ where

$\mu_A(x)$ and $v_A(x)$ are membership and non-membership function such that $\mu_A(x) , v_A(x) : X \rightarrow [0,1]$ and

$0 \leq, \mu_A(x) + v_A(x) \leq 1$ for all $x \epsilon X$.

2.2 Intutionisitic Fuzzy Number:

An Inutionistic Fuzzy subset $\overline{A} = \{ < x, \mu_A(x), v_A(x) > / x \epsilon X \}$ of the real line R is called an intutionistic fuzzy number if the following conditions hold.

* There exists mϵR such that $\mu_A(m) = 1$ and $V_A(m) = 0$

* $\mu_A(x)$ is continuous function from R $\rightarrow$[0,1] such that $0 \leq, \mu_A(x) + v_A(x) \leq 1$ for all xϵX.

2.3 Trapezoidal Intutionistic Fuzzy number:

A trapezoidal intutionistic fuzzy number is denoted by $\overline{A} = (a_1, a_2, a_3, a_4), (a_1', a_2, a_3, a_4')$ where

$a_1' \leq a_2 \leq a_3 \leq a_4'$ where the membership and non-membership function of $\overline{A}$ are in the following form

$$\mu_{\overline{A}}(x) = \begin{cases} 0 : x < a_1 or x > a_4 \\ \dfrac{(x-a_1)}{(a_2-a_1)} : a_1 \leq x \leq a_2 \\ 1 : a_2 \leq x \leq a_3 \\ \dfrac{(x-a_4)}{(a_3-a_4)} : a_3 \leq x \leq a_4 \end{cases}$$

and

$$V_{\overline{A}}(x) = \begin{cases} 0 : x < a_1 or x > a_4 \\ \dfrac{(x-a_1')}{(a_2-a_1')} : a_1' \leq x \leq a_2 \\ 1 : a_2 \leq x \leq a_3 \\ \dfrac{(x-a_4')}{(a_3-a_4')} : a_3 \leq x \leq a_4' \end{cases}$$

Accuracy function for Defuzzification:
The accuracy function of Intutionistic fuzzy number is

AF(A) = [(a+ 2(b+c) +d) + (a' +2(b'+c')+d')] / 12

3. MATHEMATICAL MODEL

3.1 Assumptions:

* The units that are transported are finally packed in parcels.

* The bio-packaging cost is incurred for each parcel altogether includes the costs of bio-packaging

and eco-labeling of the products enclosed in the parcel.

* The bio-packaging cost per parcel includes the labour cost and the material cost.
* The demand, ordering cost, proportion of the demand returned are constant per cycle.
* The proportion of waste produced per lot Q incorporates the waste produced due to the disposal of the materials used for bio-packaging.

3.2 Notations:

A order cost

c unit purchase cost

h holding cost per unit per time

P labour cost for packing per parcel

L the cost of material used for packing per parcel

a fixed cost per trip

b variable cost per unit transported per distance travelled

α proportion of demand returned ($0<\alpha<1$)

d distance travelled (from supplier to buyer, km)

D demand rate (units/year)

β social cost from vehicle emission

γ cost to dispose waste to the environment

θ proportion of waste produced per lot Q

v average velocity(km/h)

m number of parcels.

4. Crisp Model

Consider the situation where the buyer and seller are far away from each other. The items that are required by the buyer are transported to their consign via vehicles. The items may be perishable, deteriorating or non perishable. In order to categorize and to conserve the items and its quality when transported, packaging is essentially employed. The material used for packaging varies according to the of the item to be packed. The labour cost for packing also differs as per the techniques used in packaging. The units that are transported to the buyer are packed in parcels. In order to avoid the risks of the exercise of the legislations and to promote the product, the tactics of bio-packaging and eco-labeling are used. The costs of eco-labeling are included together with the costs of packaging.

A sustainable inventory model is formulated as follows.

EOQ cost per cycle : $C(Q) = A + cQ + \frac{hq^2}{2D}$

Transportation cost per cycle(delivery and collection of returned items)

$$C_t(Q) = 2a + bdQ + bd\alpha Q$$

Emission cost from transportation and package per cycle.

$$C_e(Q) = 2\beta \frac{d}{v} \text{ [The number ‘2’ refers to round trip]}$$

Waste produced by te inventory system per cycle.

$C_w(Q) = \gamma_0 + \gamma Q(\theta+\alpha)$

Bio-packaging cost per parcel includes eco-labeling, the labour costs and the material costs.

Bio-Packaging cost per parcel = P+L

The total cost of packaging per cycle $C_p(Q) = (P+L)m$

Total cost per unit of time

$$\Psi(Q) = \frac{C(Q)+C_t(Q)+C_e(Q)+C_w(Q)+C_p(Q)}{T} \quad \text{where } T = Q/D$$

$$= \frac{AD}{Q} + CD + \frac{hQ}{2} + \frac{2aD}{Q} + bdD + (1+\alpha) + \frac{2\beta dD}{vQ} + \frac{D\gamma_0}{Q} + \gamma(\theta+\alpha)D + (P+L)m\frac{D}{Q}$$

The optimal solution is $Q = \sqrt{\frac{2D[(A+2a+(P+L)m+2\beta\frac{d}{v}+\gamma_0]}{h}}$

5.Solution of the inventory model by crisp geometric programming

We solve the proposed model by applying geometric programming and the degree of difficulty is 0.

Max G(w) =

$$\prod_{i=1}^{n}\left(\frac{[A+2a+(P+L)m+\frac{2\beta d}{v}+\gamma_o]D}{Qw_{1r}}\right)^{w1r}\left(\frac{c+bd+\gamma(\theta+\alpha)D}{w_{2r}}\right)^{w2r}\left(\frac{hQ}{2w_{3r}}\right)^{w3r}\left(\frac{1+\alpha}{w_{4r}}\right)^{w4r}$$

Subject to the conditions

$w_{1r}+w_{2r}+w_{3r}+w_{4r} = 1$

$w_{1r} + w_{2r} = 0$

$- w_{1r} + w_{3r} = 0$ and $w_{4r} = 0$

Solving these conditions , we get the values of $w_{1r} = 1$, $w_{2r} = -1$ and $w_{3r} = 1$

By applying Duffin's and Peterson's theorem ,

$$\left(\frac{(A+2a+(P+L)m+\frac{2\beta d}{v}+\gamma_0)D}{Q}\right. = w_{1r}\, g(w_{1r}, w_{2r})$$

$$hQ/2 = w_{2r}\, g(w_{1r}, w_{2r})$$

$$(2D/h)(A+2a + (P+L)m + \frac{2\beta d}{v}\gamma_0 = Q^2$$

$$Q^* = \sqrt{\frac{2D[(A+2a+(P+L)m+2\beta\frac{d}{v}+\gamma_0]}{h}}$$

6. Solution of the inventory model by fuzzy geometric programming

Let AF = (a,b,c,d) be the trapezoidal fuzzy number and the objective function is

$$TC(Q) = \{A + 2a + (P+L)m + 2\beta d/v + \gamma_0\}\, D/Q + [c + bd + \gamma(\theta+\alpha)]D + hQ/2 + (1+\alpha)$$

Now we use the accuracy function of fuzzy number.

AF(A) = [(a+ 2(b+c) +d)] / 6

AF(TC(Q)) = { A + 2a + [AF(P) +AF(L)]m + 2dAF(β)/v +AF(D)/Q + [c+bd +γ(θ+α)] AF(D) + QAF(h)/2 + (1+α)

Applying Geometric Programming Technique to the solution (1)

$$G(w) = \prod_{r=1}^{n}[\{\frac{[A+2a+(AF(P)+AF(L)]m+\frac{2d}{v}AF(\beta)]AF(D)}{w_{1r}Q}\}^{w1r} * [\frac{c+bd+\gamma(\theta+\alpha)]AF(D)}{w_{2r}})]^{w2r} * [\frac{AF(h)Q}{2w_{3r}}]^{w3r} * [\frac{1+\alpha}{w_{4r}}]^{w4r}$$

Subject to the condition,

$w_{1r}+w_{2r}+w_{3r}+w_{4r} = 1$

$w_{1r} + w_{2r} = 0$

$- w_{1r} + w_{3r} = 0$ and $w_{4r} = 0$

Solving these conditions , we get the values of $w_{1r} = 1$, $w_{2r} = -1$ and $w_{3r} = 1$

By applying Duffin's and Peterson's theorem ,

$$(\frac{(A+2a+(AF(P)+AF(L))m+\frac{2AF(\beta)d}{v}+\gamma_0)AF(D)}{Q} = w_{1r}\, g(w_{1r}, w_{2r})$$

$AF(h)Q/2 = w_{2r}\, g(w_{1r}, w_{2r})$

$$(\frac{2AF(D)}{AF(h)})*(A+2a+AF(P+L)m+\frac{2AF(\beta)d}{v}+\gamma_0) = Q^2$$

$$Q^* = \sqrt{\frac{2AF(D)[(A+2a+(AF(P)+AF(L))m+2AF(\beta)\frac{d}{v}+\gamma_0]}{AF(h)}}$$

7. Solution of Inventory model by Intutionistic fuzzy Geometric Programming

Let AFI= (a,b,c,d) and (a',b',c',d') be the trapezoidal Intutionistic fuzzy number and the

objective function is TC(Q) = {A + 2a + (P+L)m +2βd/v $+\gamma_0$} D/Q +[c+ bd + γ(θ+α)]D +hQ/2 + (1+α)

Now we use the accuracy function of Intutionistic fuzzy number.

AFI(A) = [(a+ 2(b+c) +d) + (a' +2(b'+c')+d')] / 12

AFI (TC(Q)) = { A + 2a + [AFI(P) +AFI(L)]m + 2dAFI(β)/v +AFI(D)/Q + [c+bd +γ(θ+α)] AFI(D) + QAFI(h)/2 + (1+α)

Applying Geometric Programming Technique to the solution (1)

$$G(w) = \prod_{r=1}^{n}[\{\frac{[A+2a+(AFI(P)+AFI(L)]m+\frac{2d}{v}AFI(\beta)]AFI(D)}{w_{1r}Q}\}^{w1r} * [\frac{c+bd+\gamma(\theta+\alpha)]AFI(D)}{w_{2r}})]^{w2r} * [\frac{AFI(h)Q}{2w_{3r}}]^{w3r} * [\frac{1+\alpha}{w_{4r}}]^{w4r}$$

Subject to the condition,

$w_{1r}+w_{2r}+w_{3r}+w_{4r} = 1$

$w_{1r} + w_{2r} = 0$

$-w_{1r} + w_{3r} = 0$ and $w_{4r} = 0$

Solving these conditions , we get the values of $w_{1r} = 1$, $w_{2r} = -1$ and $w_{3r} = 1$

By applying Duffin's and Peterson's theorem ,

$$(\frac{(A+2a+(AFI(P)+AFI(L))m+\frac{2AFI(\beta)d}{v}+\gamma_0)AFI(D)}{Q} = w_{1r}\, g(w_{1r}, w_{2r})$$

$$AF(h)Q/2 = w_{2r}\, g(w_{1r}, w_{2r})$$

$$(\frac{2AFI(D)}{AFI(h)})*(A+2a+AFI(P+L)m+\frac{2AFI(\beta)d}{v}+\gamma_0) = Q^2$$

$$Q^* = \sqrt{\frac{2AFI(D)[(A+2a+(AFI(P)+AFI(L))m+2AFI(\beta)\frac{d}{v}+\gamma_0]}{AFI(h)}}$$

7. Numerical Example

Crisp Model:

Consider an inventory system with the following data D = 15000 units, A = \$150 , a = \$ 50, P=\$175 , L=\$40, M=5,β=\$30 , D = 500 Km , V= 100 Km/hr , h = \$ 2 , γ_0 = \$50.

The optimal order quantity is 5012 units.

Crisp Geometric Method:

The optimal order quantity is 5012 units.

Fuzzy Geometric Method:

The optimal order quantity is 5054 units.

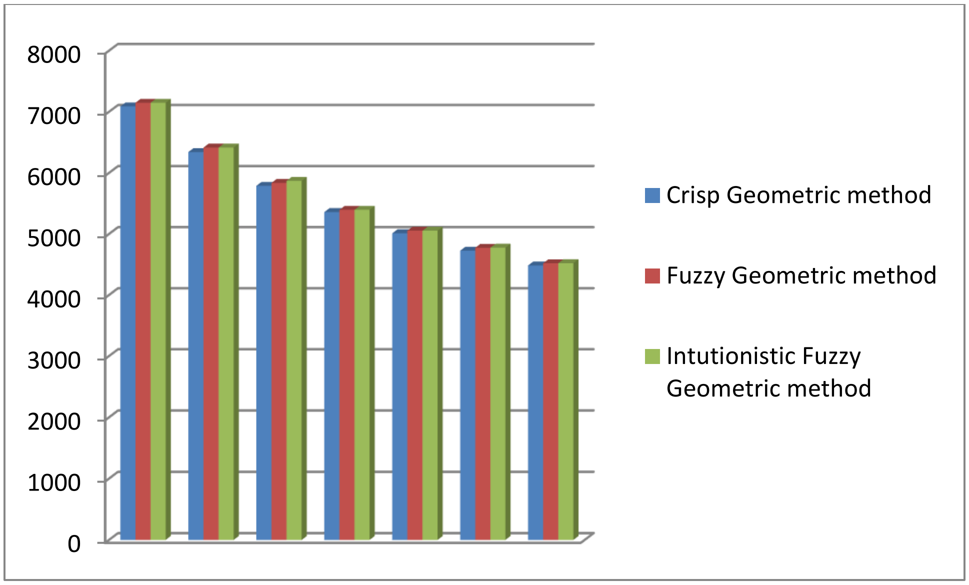
Fuzzy Intutionistic Geometric Method:

The optimal order quantity is 5054..08 units.

8. Sensitivity Analysis

In this paper, EOQ model with advantage and benefits of bio-packaging and eco-labeling of the product in intutionistic fuzzy geometric programming approach is introduced. An optimum solution is obtained by using accuracy function of trapezoidal intutionistic fuzzy number. Here we are comparing graphically , the crisp geometric, fuzzy geometric and intutionistic fuzzy geometric values as holding cost changes and hence the Intutionistic set gives the better solutions to the real world problems.

Holding cost	Crisp Geometric method	Fuzzy Geometric method	Intutionistic Fuzzy Geometric method
1	7088.72	7147.5	7147.6
1.25	6340.34	6414.5	6414.6
1.5	5787.91	5835.9	5869.3
1.75	5358.57	5395.3	5395.4
2	5012	5054.1	5054.1
2.25	4725.81	4773.6	4773.9
2.5	4483.30	4520.5	4520.6



Conclusion

In this paper an Economic order quantity model with package cost and focuses on the need of packaging , labeling of the products along with the advantages and benefits of bio-packaging and eco-labeling which speed up the product promotion in intutionistic fuzzy environment.

In recent years, several generalizations of fuzzy set theory have appeared. One of the is Intutionistic Fuzzy set, where one considers both level of membership and level of non-membership of the elements of universal set and the advantage of Geometric Programming is manifold. Geometric Programming provides us with a systematic approach for solving a nonlinear optimization problems by determining optimal values of decision variables and objection functions.

In this paper, we introduce Geometric programming in Intutionistic fuzzy environment. The decision maker is hesitating to run the process more time. This feeling must effects directly to the customer satisfaction as well as demand rate . In numerical application , we find that the optimal solution, obtained in Intutionistic Fuzzy Geometric Programming is more preferable to the decision maker than that obtained in the crisp or fuzzy geometric programming. We have performed sensitivity analysis of the proposed model and explained graphically that the holding cost increases , then the economic order quantity decreases.

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