ECONOMIC PRODUCTION QUANTITY MODEL FOR BUILDING THE QUALITY, RELIABILITY AND FLEXIBILITY OF THE PRODUCTION SYSTEM USING GEOMETRIC PROGRAMMING TECHNIQUE

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ABSTRACT: *In this paper, an economic production quantity model for building the quality, reliability and flexibility of the production system with the incorporation of the depreciation cost, maintenance cost and marketing costs. The capacity of a production system can be uncertain due to breakdowns or unplanned maintenance activity of machinery. We have considered a single objective optimization model without constraints. This objective function is imposed in fuzzy environment. Geometric programming technique provides a powerful tool for solving a imprecise optimization problems and we use nearest approximation method to convert a trapezoidal fuzzy number to an interval number and solved by geometric programming technique. Numerical example is given to illustrate the model through geometric programming method.*

KEYWORDS: Economic production quantity, fuzzy number , geometric programming method, quality, flexibility, reliability.

1. INTRODUCTION

In most of the profit maximization economic production quantity (EPQ) models, researchers considered demand as constant. This assumption however is quite impracticable in reality. It would be more realistic to consider the demand as selling price dependent, as high selling price generally makes a negative impact on a major part of the customers to buy the products. The unit production cost will be reduced if the number of units produced is large and vice-versa. Cheng [1–3] developed some inventory models with this assumption and solved them using geometric programming (GP) technique. Jung and Klein [4,5] extended the profit maximization EOQ model with the same assumptions and used GP technique.

In classical inventory models, unit-holding cost is supposed to be constant, but in reality this inventory cost is dependent on the amount produced. Hariri and Abou-el-ata [6], Abou-el-ata and Kotb [7], Abou-el-ata et al. [8] developed some inventory models with variable inventory costs and solved them by GP method. Also in the changing economic scenario the scaling factor of the unit cost may be considered as a fuzzy number rather than a constant. Again in inventory analysis it is normally assumed that the items produced are of perfect quality. However, in reality, product quality is not always perfect; it is directly affected by the capability of the production process. A high level of product quality will be achieved through substantial investment in improving the reliability of the production process. Furthermore, while the set-up time and hence set-up cost will be fixed in the short term, it will tend to decrease in the long term because of the possibility of investment in new, highly flexible machineries.

Van Beek and Putten [9] considered the issue of flexibility improvement in production /inventory management under various scenarios. Some researchers have considered the issues of process reliability, quality improvement and set-up cost reduction in EOQ problems like [1,10–14]. Recently [15] has developed an inventory model considering flexibility and reliability consideration of production process. Duffin et al. [16] first showed that the GP technique could be used with some advantages for optimization problems of a particular type. Later [1,6,7,17,18] solved some inventory problems using GP method. Cao [19,20] introduced fuzzy geometric programming which was later used by other researchers in solving fuzzy decision making problems.

Roy and Maiti [25] solved the classical EOQ model for a single item in fuzzy environment. Roy and Maiti [26] also studied the fuzzy EOQ model with demand-dependent unit price and imprecise storage area by both fuzzy GP and NLP method. Mandal et al. [27] discussed a multi-item fuzzy inventory problem with three constraints and solved by GP technique. Mandal and Roy [28] formulated an inventory model with stock dependent inventory costs that they have solved by GP technique.

In this paper, an economic production quantity model for building the quality, reliability and flexibility of the production system with the incorporation of the depreciation cost, maintenance cost and marketing costs. The capacity of a production system can be uncertain due to breakdowns or unplanned maintenance activity of machinery. For solving a imprecise optimization problems, we use nearest approximation method to convert a trapezoidal fuzzy number to an interval number and solved by geometric programming technique.

2. Mathematical model

2.1 Assumptions

1. Production rate and demand rate are constant (P > D).

2. Replenishment is instantaneous.

3. No excess stock is carried and no shortage is allowed.

4. The production quantity is produced in batches

5. Depreciation is the function of use.

2.2 Notations

D demand per unit of time

P production per unit of time

x D/ P

1-x the fraction of time the production process spends actually idling

A fixed ordering cost/ set up cost per production run

h holding cost per unit per unit of time.

C unit production cost

p_i proportion of cost of machine mi (i = 1, . . . , n)depreciated

Ci cost of the machine mi (i = 1, . . . , n)

fi fixed maintenance cost of machine mi (i = 1, . . . , n)

ni variable maintenance cost of machine mi (i = 1, . . . , n)

Mi volume of investments in marketing method i = 1, . . . , k per unit time

T cycle length.

3. Crisp Model

Consider a manufacturer who produces a single product by employing highly advanced machinery. As the production process is carried on in full swing to satisfy the customers‟ demands the machines are subjected to continuous working which starts to depreciate over a period of time. To avoid the machine breakdown, the manufacturers maintain the quality of the machines by periodical inspection, repairing the worn out parts of the machine with the assistance of skilled labours, tooling, machine cleaning and so on. The expenditure of the manufacturer not only includes set up cost, production costs and holding costs , but also it comprises of expenditure spent for maintaining high degree of quality, reliability and flexibility which in turn incorporate the costs of depreciation, maintenance and marketing.

A mathematical model is formulated as follows :

The EPQ cost per unit of time AD/Q + hQ(1 x) / 2

The production cost per cycle Cp(Q) = cQ

The Depreciation cost per cycle:

Suppose that the manufacturer owns machine mi (i = 1, . . . , n) to execute sequentially the various tasks involved in the production process. It is quite natural for the machine to get degraded due to continuous wear and tear, so the value of the machines gets depreciated. The proportion (pi) of cost of machine mi (i = 1, . . . , n) depreciated is determined by various methods and it varies from one machine to another. Therefore the depreciated cost per cycle is as follows : C_D (Q) = $\sum_{i=1}^{n} p_i C_i$ = L (say)

The Maintenance Cost per Cycle:

To maintain the degree of reliability and to prevent machine breakdown the machines have to be properly taken care. Since each machine serves different purpose the pattern of its maintenance also differs. Therefore the maintenance cost per cycle is as follows : C_M(Q) = $\sum_{i=1}^{n} (f_i + n_i)$ = F (say)

The Marketing Cost per Cycle :

To propagate the product in the market rapidly the manufacturer prefers marketing strategies, for which cost is incurred. C_A(Q) = $\sum_{i=1}^{n} M_i$ = S (say)

Total Cost per Unit of Time TC(Q) = $\frac{AD}{Q} + \frac{hQ(1-x)}{2} + cD + \frac{D}{Q}(L + F + S)$

Optimum solution is $Q = \sqrt{\frac{2D(A+L+F+S)}{h(1-x)}}$ -----------------(*)

4. Fuzzy Model

Total cost per unit of time : $TC(Q) = \frac{A\overline{D}}{Q} + \frac{\overline{h}Q(1-x)}{2} + c\overline{D} + \frac{\overline{D}}{Q}(\overline{L}+\overline{F}+\overline{S})$

5. Geometric programming problem

Primal problem: Primal Geometric Programming(PGP) problem is

$$\text{Minimize } g_0(t) = \sum_{k=1}^{T_0} C_{0k} \prod_{j=1}^{m} t_j^{\alpha_{0kj}}$$

Subject to $\sum_{k=1}^{T_0} C_{rk} \prod_{j=1}^{m} t_j^{\alpha_{rkj}} \leq 1$, (r= 1,2,…..l) , j= (1,2,3…..m) $t_j>0$

Where $C_{0k} > 0$ (k=1,2,…T_0) C_{rk} and α_{rk} are real numbers. It is constrained polynomial geometric problem. The number of term each polynomial constrained functions varies and it is denoted by T_r for each r= 0,1,2….. Let T= $T_0+T_1+T_2+\ldots\ldots+T_l$ be the total number of terms in the primal program. The Degree of difficulty is (DD) = T – (m+1)

Dual Problem:

$$\text{Maximize } = \prod_{r=0}^{l}\prod_{k=1}^{T^r}\left(\frac{C_{rk}}{\delta_{rk}}\right)^{\delta rk}\left(\sum_{s=1+T_{r+1}}^{T}(\delta_{rs})^{\delta rk}\right)$$

Subject to $\sum_{k=1}^{T_0} \delta_{0k} = 1$ (Normality condition)

$\sum_{r=0}^{l}\sum_{k=1}^{T_r} \alpha_{rkj}\delta_{rk} = 0$ (Orthogonality conditions)

$\delta_{rk} > 0$, (Positive constant)

6. Solution procedure of crisp model by geometric programming technique

Here the primal problem is Min $TC(Q) = \frac{(A+L+F+S)D}{Q} + \frac{hQ(1-x)}{2} + cD$ ---------(1)

Corresponding dual form of (1) is given by

$$\text{Max } G(w) = \prod_{r=1}^{n} \left(\frac{(A+L+F+S)D}{Qw_{1r}}\right)^{w1r} \left(\frac{hQ(1-x)}{2w_{2r}}\right)^{w_{2r}} \left(\frac{CD}{w_{3r}}\right)^{w_{3r}} \text{-----(2)}$$

Subject to $w_{1r}+w_{2r}+w_{3r} = 1$ ....(3)

$-w_{1r} + w_{2r} = 0$ ....(4)

$w_{1r} + w_{3r} = 0$ ....(5)

Solving the equations 3, 4 ,5 , we get $w_{2r} = w_{1r} = 1$ and $w_{3r} = -1$

Putting the values in (2) we get the optimal solution of dual problem and Q is obtained by using the primal dual relation as follows. We apply Duffin and Pelirson's theorem, of geometric problem.

$(A+L+F+S)D \,/\, Q = w_{1r}^{*}\ g(w_{1r}^{*}, w_{2r}^{*})$ ....(6)

$hQ(1-x)/2 = w_{2r}^{*}\ g(w_{1r}^{*}, w_{2r}^{*})$ ....(7)

From (6) and (7) $Q = \sqrt{\dfrac{2D(A+L+F+S)}{h(1-x)} \dfrac{w^{*}_{2r}}{w^{*}_{1r}}}$

Let $w_{1r}^{*} = w_{2r}^{*} = ½$ and $w_{3r}^{*} = 0$

We get $Q = \sqrt{\dfrac{2D(A+L+F+S)}{h(1-x)}}$

7. Nearest interval approximation

Suppose $\overline{A}$ is a fuzzy number with α cut is $[A_L(\alpha), A_R(\alpha)]$. Then

The interval $C_d(\overline{A}) = [\int_0^1 [A_L(\alpha)\, d\alpha, \int_0^1 A_R(\alpha) d\alpha]$ with respect to the metric d . Let $\overline{A} =$ (a,b,c,d) be a trapezoidal fuzzy number. The α – cut interval of $\overline{A}$ is defined as $[A_L(\alpha), A_R(\alpha)]$ where

$A_L(\alpha) = a + (b-a)\alpha$ and $A_R(\alpha) = d - (d-c)\alpha$. By nearest interval approximation method the lower limit of the interval is $C_L = \dfrac{a+b}{2}$ and upper limit of the interval is $C_R = \dfrac{c+d}{2}$.

8. Solution procedure of Fuzzy model by geometric programming technique

When $\overline{D}$ = (D_1, D_2, D_3 , D_4) , $\overline{H}$ = (H_1, H_2, H_3, H_4), $\overline{P}$ =(P_1, P_2, P_3 ,P_4), $\overline{L}$ =(L_1 ,L_2, L_3, L_4) , $\overline{F}$ =(F_1,F_2,F_3, F_4) $\overline{S}$ = (S_1, S_2, S_3, S_4) are trapezoidal fuzzy number, then the fuzzy model is

$$\text{Min} \quad TC(Q) = \frac{(A+\overline{L}+\overline{F}+\overline{S})\overline{D}}{Q} + \frac{\overline{h}Q(1-x)}{2} + c\overline{D} \qquad \text{---------------(8)}$$

Using the nearest interval approximation method, the interval number corresponding trapezoidal number

$\overline{D}$ = (D_1, D_2, D_3, D_4) = [D_L , D_R] = [(D_1+D_2)/2 , (D_3+D_4)/2]

$\overline{H}$ = (H_1, H_2, H_3, H_4) = [H_L , H_R] = [(H_1+H_2)/2 , (H_3+H_4)/2]

$\overline{P}$ = (P_1, P_2, P_3, P_4) = [P_L , P_R] = [(P_1+P_2)/2 , (P_3+P_4)/2]

$\overline{L}$ = (L_1, L_2, L_3, L_4) = [L_L , L_R] = [(L_1+L_2)/2 , (L_3+L_4)/2]

$\overline{F}$ = (F_1, F_2, F_3, F_4) = [F_L , F_R] = [(F_1+F_2)/2 , (F_3+F_4)/2]

$\overline{S}$ = (S_1, S_2, S_3, S_4) = [S_L , S_R] = [(S_1+S_2)/2 , (S_3+S_4)/2]

And the problem (8) reduces to Min TC(Q) =

$$\frac{\{A+[L_L,L_R]+[F_L,F_R]+[S_L,S_R]\}[D_L,D_R]}{Q} + \frac{[h_L,h_R]Q(1-x)}{2} + C[D_L,D_R]$$

And the dual form is

$$\text{Max } G^*(w) = \prod_{r=1}^{n}\left(\frac{(A+\overline{L}+\overline{F}+\overline{S})\overline{D}}{Qw_{1r}}\right)^{w1r}\left(\frac{\overline{h}Q(1-x)}{2w_{2r}}\right)^{w_{2r}}\left(\frac{C\overline{D}}{w_{3r}}\right)^{w_{3r}} \qquad \text{-------------(9)}$$

Subject to $w_{1r}+w_{2r}+w_{3r} = 1$ …………….(10)

$-w_{1r} + w_{2r} = 0$ …………….(11)

$w_{1r} + w_{3r} = 0$ ……………….(12)

Solving the equations (10), (11) ,(12) , we get w_{2r} = w_{1r} =1 and w_{3r} = - 1

Putting the values in (8) we get the optimal solution of dual problem and Q^* is obtained by using the primal dual relation as follows. We apply Duffin and Peterson's theorem of geometric problem.

$((A+\bar{L}+\bar{F}+\bar{S})\bar{D}$ / Q = $w_{1r}^* \; g(w_{1r}^*, w_{2r}^*)$ …………………..(13)

$\bar{h}$ Q(1-x) /2 = $w_{2r}^* \; g(w_{1r}^*, w_{2r}^*)$ …………………………………..(14)

From (13) and (14) $Q^* = \sqrt{\dfrac{2(D_L, D_R)(A+(L_L, L_R)+(F_L, F_R)+(S_L, S_R))}{(h_L, h_R)(1-x)}} \dfrac{w^*_{2r}}{w^*_{1r}}$

Let $w_{1r}^* = w_{2r}^* = ½$ and $w_{3r}^* = 0$

We get $Q^* = \sqrt{\dfrac{2(D_L, D_R)(A+(L_L, L_R)+(F_L, F_R)+(S_L, S_R)}{(h_L, h_R)(1-x)}}$ ---------(15)

And $Q^{*L} = \sqrt{\dfrac{2D_L(A+L_L+F_L+S_L)}{h_R(1-\dfrac{D_R}{P_R})}}$ and $Q^{*R} = \sqrt{\dfrac{2D_R(A+L_R+F_R+S_R)}{h_L(1-\dfrac{D_L}{P_L})}}$ -----

--(16)

9. NUMERICAL EXAMPLE

For crisp model:

Consider an inventory problem with the following data .

P = production rate = 1000 units/time

D= Demand = 900 units/time

A = set up cost = \$100 and h= \$1.5.

Suppose the manufacturing firm consists of three machines then the required data are as follows:

p_1 = 2% , p_2= 3% , p_3 = 1%, C_1= \$20000, C_2 = \$ 30000 , C_3 = \$35000, f_1 = \$100 , f_2 = \$150 , f_3 =\$190,

n_1= \$50 , n_2 = \$75, n_3=\$90, M_1= \$850, M_2 =\$900

Then the optimal quantity obtained by using (*) is 7061.

Fuzzy model:

$\overline{D}$ = (300 , 600, 1200, 1500) , $\overline{L}$ =(1450,1550,1750,1850) $\overline{F}$ = (455, 555, 755, 855)

$\overline{S}$ =(1550,1650,1850,1950) $\overline{h}$ =(0.5,1 ,2, 2.5) $\overline{P}$ =(400,700,1300,1600)

(D_L, D_R) = (450,1350) , (L_L, L_R) = (1500,1800) ,(F_L, F_R) = (505, 805) , (S_L, S_R) = (1600, 1900)

(h_L, h_R) = (0.75 , 2.25) ,(P_L, P_R) = (550,1450)

Then the optimal quantity is obtained by using (15) and (16) is

$\mathbf{Q^{*L} = 4635.606}$, $\mathbf{Q^{*R} = 9548.769}$ **and** $\mathbf{Q^{*} = 7092.1}$

10.SENSITIVITY ANALYSIS:

The sensitivity analysis is performed for checking the effectiveness of the economic production quantity model with quality, reliability and flexibility using Geometric programming technique. The marketing strategies are purely customer based help in retaining the customers by satisfying their needs.

Conclusion

In this paper, a fuzzy economic production quantity model with building for reliability, flexibility and quality of the production system using geometric programming technique is obtained and this will increase the total profit. It is concluded that the value of optimal production quantity is much sensitive with respect to the maintenance cost, depreciation cost and marketing cost and so the fuzzy model permits the flexibility in the system.

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